Name:

Activity 3: Transition from Expressions to Equations

**Part I (with CAS): Introduction to the use of the SOLVE command**

Recall in our first activity on equivalence of expressions that we encountered expressions that were not equivalent (a reminder of the definition of equivalence: “If, for any admissible number that replaces *x*, each of the expressions gives the same value, we say that these expressions are equivalent on the set of admissible values.”).

With those non-equivalent expressions, when we entered into the CAS the equations formed with such expressions, the CAS did not display “true”. This was because there are only *some* (or *no*) values of x that, when substituted into both sides of the equation, produce equal results. In the present activity we will use the CAS to find the values of x that produce equal results for given expressions.

Here is an example of two expressions that are clearly not equivalent: *x*2 and *x.*

If we enter into the calculator an equation formed of these two expressions (*x*2= *x*), it will therefore not display “true”. To find those values of x for which the two expressions produce equal values, we can use the SOLVE command of the CAS.

**Syntax**: SOLVE (Expr1 = Expr2, *x*), presuming that “*x*” is the variable name that appears in each expression, and Expr1 and Expr2 represent the given expressions.

**Solve the equation *x*2= *x* using the SOLVE command in your CAS**.

1. What does the CAS display as a result?

*x = 1 or x = 0*

1. Anticipate what the calculator would display were you to substitute each of these *x*-values back into the equation?

True

1. Using your CAS’s “with operator” (“ **|** ”), verify that the calculator indeed

displays what you expected in question 2.

**Syntax**: Expr1=Expr2 **|** *x* = *value*

**Terminology**: The values of x for which both expressions produce equal results are commonly referred to as “solutions” to the equation.

**Part II (with CAS):**

**Expressions revisited, and their subsequent integration into equations**

Here are three expressions:

1. *x(x2-9)*
2. *(x+3)(x2-3x)-3x-3*
3. *(x2-3x)(x+3)*

II(A) Use your calculator to determine which of these expressions are equivalent. Fill in the table below with the appropriate information.

|  |  |  |
| --- | --- | --- |
| What I entered into the CAS | What the CAS displays | My interpretation of what the CAS displays |
| *x(x2-9) = (x+3)(x2-3x)-3x-3* | *x(x2-9) = x3—12x—3* | Exp1 and Exp2 are not equivalent |
| *x(x2-9) = (x2-3x)(x+3)* | true | Exp1 and Exp3 are equivalent |
|  |  |  |

II(B) Which expressions are equivalent? Which are not equivalent? Please explain.

Exp1Exp3 (see table above), and

Exp1 and Exp2 are not equivalent (see table above).

Therefore, Exp2 and Exp3 are not equivalent (by transitivity of equivalence).

II(C) Construct an equation using one pair of the given expressions that are not equivalent (see Question IIB, above). Use your calculator to determine those values of *x*, if any, for which both expressions in your equation are equal.

|  |  |
| --- | --- |
| What I entered into the CAS | What the CAS displays |
| Solve (*x(x2-9) = (x+3)(x2-3x)-3x-3, x*) | *x = -1* |
|  |  |

II(D) How would you use the CAS to verify that the values you found for x (in Question IIC, above) are solutions to your equation? Fill in the table below with the appropriate information.

|  |  |
| --- | --- |
| What I would enter into the CAS | The result that the CAS would display |
| *x(x2-9) = (x+3)(x2-3x)-3x-3* **|** *x = -1* | true |

II(E) Construct an equation from another pair of the given expressions that are **not equivalent** (see Question IIB, above). Without using the CAS and without using paper and pencil algebra (use only a logical argument), determine the solution(s) to this equation. Please explain.

*(x2-3x)(x+3) = (x+3)(x2-3x)-3x-3*

The solution to this equation has to be *x = -1*

Explanation:

In IIC above, the solution to the equation *x(x2-9) = (x+3)(x2-3x)-3x-3* was seen to be *x=–1*. The equation in the present question (IIE) has the same expression on the right side as does the equation of IIC. Furthermore, the left side of the equation in IIC was found to be equivalent (in IIA) to the left side of the equation in the present question. In other words, the left-hand-side expression of the equation in IIE is simply another form of the left-hand-side expression of the equation in IIC. Both equations must thus have the same solution (as well, there are no inadmissible values of *x* in these two equations that would constrain this claim).

II(F) Construct an equation using a pair of the given expressions that **are equivalent** (see Part II B, above). Without using your CAS and without using paper and pencil algebra (use only a logical argument), determine the solution(s) of this equation. Please explain.

*x(x2-9) = (x2-3x)(x+3)*

Since these two expressions are equivalent, this equation is, by definition of equivalence, identically true. That is, every real number is a solution. [Incidentally, such equations are called “identities.”]

Another way of explaining this is to say that either expression in this equation is merely a re-expression of the other. Since there are no restrictions on the values that *x* can assume in either expression, the equation just describes two ways to re-express a common set of operations on those values, regardless of what those values may be.

# Classroom discussion of Parts I and II

**Part III (paper & pencil): Constructing equations and identities**

**III(A)** 1. Construct an equation made from two equivalent expressions of your own choosing.

*2x(x + 3) = 2x2 + 6x*

2. Explain your reasons for choosing these two particular expressions.

Distributivity of multiplication over addition always produces equivalent expressions.

3. Without solving it, what can you say about the solutions of this equation?

The solution set consists of all real numbers.

4. How would you use the CAS to support your response to Question A3 just above?

I would employ a test of equality: after entering this equation, I would expect the CAS to display “true”.

**III(B)** 1. Construct an equation made from two non-equivalent expressions of your own choosing.

*x = x + 1*

2. Explain your reasons for choosing these two particular expressions.

No number, *x,* is ever equal to one more than itself, *x + 1*.

This ensures that the two expressions are not re-expressible in a common form.

3. What can you say about the solutions of this equation?

This particular equation has no solutions because it is a contradiction, as implied in the response to Part B2 above.

4. How would you use the CAS to support your response to Question B3 just above?

I would enter “Solve(*x = x + 1*, *x*)”, and expect the CAS to display “false”.

Classroom discussion of Part III, A and B

Part IV (with CAS): Synthesis of various equation types

1. Solve the following equations using the SOLVE command of the CAS.

|  |  |
| --- | --- |
| Given equation | What the CAS displays |
| 1. *(2–x)2 = x(2x–4)* | *x = 2* or *x = -2* |
| 2. *(x–5)(3x+7)–5 = 3x2-8x–40* | true |
| 3. *3x2–x–1 = 2x+5* | *x = 2* or *x = -1* |
| 4. | false |

2. How do you interpret each of the CAS displays in Question 1 above?

Equation 1 has two solutions; in other words, there are two real numbers (*x = 2* and *x =2)* for which the expressions *(2–x)2* and *x(2x–4)* produce equal results.

Equation 2 is true for all real numbers; in other words, the left-hand expression is equivalent to the right hand expression.

Equation 3 has two solutions; in other words, there are two real numbers (*x = 2* and *x =1)* for which the expressions *3x2–x–1* and *2x+5* produce equal results.

Equation 4 has no solutions; in other words, there are no real numbers for which the left-hand expression and the right-hand expression produce equal results.

3. What does the nature of an equation’s solution(s) indicate about the equivalence or non-equivalence of the expressions that form the equation?

If an equation is true for all real numbers, the left-hand expression of the given equation is equivalent to the right-hand expression. In other words, for each real value of x (except for the inadmissible values), the left-hand result will equal the right-hand result. In all other cases, the left-hand expression of the given equation is not equivalent to the right-hand expression. For those cases, there may be certain values of x (those values that are the solutions to the equation) for which the left-hand side is equal to the right-hand side (even if the two expressions on each side are not equivalent to each other).

Classroom discussion of Part IV