Name: Date:

Activity 6: Factoring

# Part I (Paper & pencil, and CAS): Seeing patterns in factors

1. (a) Before using your calculator, try to recall the factorization of each algebraic expression listed in the left column of this table:

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| --- | --- |
| Factorization using paper and pencil | Verification using FACTOR (show result displayed by the CAS) |
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|  |  |
|  |  |
|  |  |

**Classroom discussion of Part I, 1a**

1. (b) Perform the indicated operations (using paper and pencil)

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2. (a) Without doing any algebraic manipulation, anticipate the result of the following product:

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2. (b) Verify the anticipated result above using paper and pencil (in the box below), and then using the calculator.

2. (c) What do the following three expressions have in common? And, also, how do they differ?

, , and .

2. (d) How do you explain the fact that the following products result in a binomial: two binomials, a binomial with a trinomial, and a binomial with a quadrinomial?

**Classroom discussion following Question 2d**

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2. (e) On the basis of the expressions we have found so far, predict a factorization of the expression .

2. (f) Explain why the product (*x* –1) (*x*15 + *x*14 + *x*13 + … + *x*2 + *x* + *1*) gives the result *x*16–*1* ?

2. (g) Is your explanation (in (f), above) also valid for the following equality:

(*x* –*1*) (*x*134 + *x*133 + *x*132 + … + *x*2 + *x* + *1*) = *x*135–*1* ?

Explain:

Classroom discussion of Part I, #1 #2

## Part II: Toward a generalization (activity with paper & pencil and with calculator)

II(A) 1. In this activity each line of the table below must be filled in completely (all three cells), one row at a time. Start from the top

row (the cells of the three columns) and work your way down.

If, for a given row, the results in the left and middle columns differ, reconcile the two by using algebraic manipulations in the right hand column.

|  |  |  |
| --- | --- | --- |
| Factorization using paper and pencil | Result produced by FACTOR command | Calculation to reconcile the two, if necessary |
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II.(A).2. Conjecture, in general, for what numbers *n* will the factorization of :

1. contain exactly two factors?
2. contain more than two factors?
3. include  as a factor?

Please explain:

**Classroom discussion of Part II A**

**Part II continued (with paper and pencil, and with calculator)**

II(B) 1. As with Part A above, each line of the table below must be filled in completely (all three cells), one row at a time before proceeding to the next row. Start from the top row and work your way down.

If, for a given row, the results in the left and middle columns differ, reconcile the two by using algebraic manipulations in the right hand column.

|  |  |  |
| --- | --- | --- |
| Factorization using paper and pencil | Result produced by FACTOR command | Calculation to reconcile the two, if necessary |
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II.(B).2. On the basis of patterns you observe in the table II.B above, revise (if necessary) your conjecture from Part A. That is, for what numbers *n* will the factorization of :

i) contain exactly two factors?

ii) contain more than two factors?

ii) include  as a factor?

Please explain:

II(C) Without using your calculator, answer the following questions:

1. Does 

* 1. contain more than two factors?
  2. include  as a factor?

Please explain:

2. Does 

1. contain more than two factors?

ii) include  as a factor?

Please explain:

3. Does 

1. contain more than two factors?

ii) include  as a factor

Please explain:

**Classroom discussion of Part II B and C**

**Part III: Challenge**

Explain why (*x* + 1) is always a factor of  for even values of *n* ≥ 2.

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