Title: Researcher-designed resources and their adaptation within classroom teaching practice: Shaping both the implicit and the explicit

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Abstract: Little is known about the ways in which teachers take on research-based resources and adapt them to their own needs. Teachers' adaptation of researcher-designed resources is a new area of research, but one that fits into the emerging *documentational approach of didactics.* This study focused on the manner in which three teachers who were participating in a research project on the learning of algebra with CAS technology spontaneously adapted the resources that were designed specifically by the researchers for use in the project. Analysis of the classroom-based observations of teaching practice showed that adaptive shaping occurred with respect to all three key features (the mathematics, the students, and the technology) of our researcher-designed resources, whether our intentions with respect to those features were explicitly stated or implicitly suggested. The study also found that embarking with the same set of task-sequences, and sharing the same goal of participating in a research project aimed at developing the technical and theoretical knowledge of algebra students within a CAS-supported environment, can lead to quite different uses of the resources. The teachers brought into the study their own beliefs, knowledge, and customary ways of interacting with their students. Quite clearly this had an impact for each class on the nature of the mathematical activity engaged in.

Keywords

Researcher-designed resources, teachers' adaptations of resources, implicit and explicit aspects of teaching resources, teaching with CAS tools, teachers' beliefs and goals, algebra teaching, teaching practice, underpinnings of teachers' adaptations, stability of teachers' practice, pedagogical contract with students.

Chap. 10 – Researcher-designed resources and their adaptation within classroom teaching practice: Shaping both the implicit and the explicit

10.1 introduction

Mathematics education research has, over the years, yielded numerous resources, many of which have been both designed with the practitioner in mind and made accessible to them. But little is known about the ways in which teachers take on such research-based resources and adapt them to their own needs. In 2000, Adler proposed that, “mathematics teacher education needs to focus more attention on resources, on what they are and how they work as an extension of the teacher in school mathematics practice” (Adler, 2000, p. 205). However, much of the resource-related research has focused more on the mathematical design of the resources (e.g., Ainley & Pratt, 2005) or on their impact with respect to student learning (e.g., Hershkowitz et al., 2002), rather than on the ways in which the resources are used by teachers and on why they are used in these ways. The research presented in this chapter centres on the ‘how’ and the ‘why’ of teachers’ adapting of researcher-designed resources.

10.1.1 The literature that this research draws upon

The ways in which teachers adapt researcher-designed resources – with the emphasis here on the fact that the resources are researcher-designed rather than, for example, commercially-based – is a new area of research, but one that fits into the recently emerging frame that is being referred to as the *documentational approach of didactics* (Gueudet & Trouche, 2009). Gueudet and Trouche (Chapter 2) state that the documentational approach focuses on both the interactions between mathematics teachers and resources and the ways in which these interactions are constituent of professional growth. They use the term *resources* to include a variety of artefacts: textbook, piece of software, student sheet, discussion with a colleague or with students. While the long-term evolutionary process of teachers’ choosing, transforming, implementing, and revising of resources is considered central for Gueudet and Trouche, they do admit that “whatever the time scale, the integration and appropriation of new resources is a complex issue” (Gueudet & Trouche, 2009, p. 212).

Some of the past research on teaching, even if it has been built upon different theoretical frames and structures, deals with teachers’ day-to-day practice and their shaping of curricular resources, and is thus pertinent to the research to be described in this chapter. A rich research base exists with respect to what teachers do in their day-to-day practice (and why), focusing on, for example, the roles played by teachers' knowledge base (e.g., Borko & Putnam, 1996; Fennema & Franke, 1992; Shulman, 1986) and their beliefs (Thompson, 1992). More recently, Robert and Rogalski (2005) have argued that the factors underlying teaching practice include each teacher's personal history, experience and professional history in a given activity, and knowledge and beliefs about mathematics and teaching. Sensevy et al. (2005) have noted that the didactic techniques they observed within each of the teacher's teaching of the same content were "produced on a background of beliefs" (p. 174) that gave rise to a certain consistency in the practice of each teacher. Such consistency constitutes that which is sometimes referred to in the literature as *teaching style* (e.g., Evans, Harkins, & Young, 2008).

In that the researcher-designed resources central to this chapter involve the use of technological tools and novel tasks, the existing literature related to how teachers adapt newer reform-based curricular material, as well as the studies on teachers' practice involving digital resources and related task materials, is of interest. Research suggests that different teachers enact the same curriculum materials differently (e.g., Chavez, 2003) and, moreover, that the same teacher enacts the same curriculum materials differently in different classes (Eisenmann & Even, in press). Remillard (2005) carried out a review of the research literature on teachers' use of mathematical curricula, in particular, research pertaining to reform-based curricula. She found that, although such studies offer insights into the influences underlying curriculum use, they provide little clarity on how teachers interact with curricular resources (note that Remillard's Chapter 6 in this volume focuses specifically on this gap in the literature). In addition, research studies have been carried out with the goal of developing design principles for the creation of curriculum materials that are supportive of teacher learning in various curricular areas (e.g., Davis & Krajcik, 2005; Ruthven et al., 2010); however, the aim of this chapter is to focus more directly on what teachers actually do when attempting to integrate novel, researcher-designed, materials into their classroom practice.

Schoenfeld (1998) has developed a theoretical model of how and why teachers do what they do while engaged in the act of teaching. The model describes the ways in which teachers' goals, beliefs, and knowledge interact, and accounts for teachers' moment-to-moment decision-making and actions. While Schoenfeld's model does not treat teachers' practice with novel materials, one of the cases that he discusses in detail is that of a relatively new teacher whose experience with the given curricular material was limited. This allows us to infer that, according to Schoenfeld, the practice of teachers with novel material, whether it be the activity of the lesson planned by the teacher, or the unscripted activity engendered by unexpected students' difficulties or responses, is regulated by deep-seated goals, beliefs, and knowledge.

In contrast to attributing teaching practice to goals, beliefs, and knowledge, some of the recent research involving the integration of computer-technology resources situates such practice within Saxe's (1991) frame of emergent goals. Within this frame, Lagrange and Monaghan (2010) argue that models that focus on teachers' established routines (as described, e.g., by Robert & Rogalski, 2005) are insufficient for analyzing teachers' activities in technology-based lessons. For Lagrange and Monaghan, a central question is why it is difficult, even for experienced teachers, to develop consistent activity when using technology. They posit that teachers' practices in dealing with the complexity of classroom use of technology are far from stable. Drijvers et al. (2010), who have combined the constructs of emergent goals (Saxe, 1991) and instrumental orchestration (Trouche, 2004; see also Drijvers, Chapter 14), suggest that the unstable practices of teachers within technology environments might still be rooted within a system of more stable beliefs and knowledge.

Remillard (2005) points out that, within the studies on teacher characteristics that influence curriculum use, knowledge and beliefs are the most studied. However, she also argues that features of the curriculum matter to curriculum use as much as characteristics of the teacher and that such research is rather unexplored terrain: "While it is common for studies of teachers' curriculum use to delve deeply into individual teachers' resources and characteristics, it is less common for researchers to examine use through the analyses of the structures and features of the curriculum" (p. 235). She signals a need for more research in this area, focusing in particular on the participatory relation between teacher and curriculum. In the spirit of Remillard's recommendation, this chapter uses the main features of the resources designed by the research team as a backdrop for analyzing both the adaptations brought to the researcher-designed resources by participating teachers within their teaching practice, as well as the relation of these adaptations to individual teacher characteristics.

10.2 Background

The study presented herein is part of a larger program of research, the first phase of which was oriented toward student learning: its central objective was to shed light on the co-emergence of algebraic technique and theory within an environment involving novel tasks and a combination of Computer Algebra System (CAS) and paper-and-pencil technologies (see Kieran & Drijvers, 2006). The second phase of the program, which was oriented toward teaching practice, included secondary analyses of the video-data from the first phase. From the start, these analyses disclosed specific differences in the manner in which teachers were interpreting the researcher-designed tasks in their day-to-day practice. Individual teachers were mediating the technical and theoretical demands of the tasks for their students in quite different ways. These secondary analyses provide the foundation for this chapter.

10.2.1 The three teachers and their students

Of the five teachers participating in our initial study, the three who are featured in this chapter were selected because they all taught in the same city – a large urban metropolis – and thus shared a certain common curricular experience. They shall be named T1, T2, and T3. To help in further maintaining their anonymity, the masculine gender will be used throughout.

T1, whose undergraduate degree was in economics, had been teaching mathematics for five years, but had not had a great deal of experience with technology use in mathematics teaching, except for the graphing calculator. In observing T1's teaching prior to the start of the research, the researchers noted that he encouraged his pupils to talk about their mathematics. T1's class of Grade 10 students was considered by the teacher to be of medium-high mathematical ability. They were quite skilled in algebraic manipulation, as was borne out by the results of a pretest we administered. They were used to handling graphing-calculator technology on a regular basis, but had not experienced CAS technology prior to the start of the research.

T2, who was the most mathematically qualified of the three teachers, had previously taught college-level mathematics before teaching at the secondary school. He had taught mathematics for 16 years, half of this time at the college level. T2 was a leader with respect to the advancement of the use of technology in the school where he was teaching. In our pre-study observations of his teaching practice, we noted that his teaching style tended to be teacher-centred. T2's students seemed very strong, mathematically-speaking, based on the same pretest as was mentioned above, and were experienced with the various capabilities of graphing-calculator technology, but not with CAS.

T3, whose undergraduate degree was in the teaching of high school mathematics, had five years of experience in the teaching of mathematics at the secondary level. While he had some prior experience with the use of graphing-calculator technology in his teaching, he had never before used CAS. T3's students were considered by their teacher to be of average mathematical ability. They had some graphing-calculator experience, but none with CAS. The pretest that we administered indicated that they were weaker in symbol-manipulation ability than the students of the other two classes.

10.2.2 Methodological aspects

At the same time that our research team began to create the task-sequences that would encourage both technical and theoretical development (see also Chevallard, 1999, Artigue, 2002, and Lagrange, 2003, for more on *task*, *technique*, and *theory*) in 10th grade algebra students – a creation process that took well over a year – we also made contact with several practicing mathematics teachers to see if they might be interested in collaborating with us. The form of collaboration that we arranged was on several levels. First, the teachers were our practitioner-experts who, within a workshop setting, provided us with feedback regarding the nature of the tasks that we were conceptualizing. They also spent some time learning how to use the CAS technology (hand-held TI-92 Plus calculators – the same devices that would be lent to the students for the entire school year). As well, the week-long workshop included discussions related to the main mathematics-related and technology-related intentions of the researcher-designers. Second, after modifying the task-sequences in the light of the teachers' feedback, we requested that, at the beginning of the following semester, they integrate all of the task-sequences into their regular mathematics teaching and that they be willing to have us act as observers in their classrooms. Third, throughout the course of our classroom observations, which occurred over a five-month period in each class, we also offered a form of ongoing support to the participating teachers by being available to discuss with them whatever concerns they might have. In addition, we conducted interviews with some of them immediately after certain lessons that we had perceived to be worthy of further conversation.

In contrast to the methodological approach adopted by Gueudet and Trouche (Chapter 2), which includes consideration of teachers' work outside of class, their existing materials, and how their use of these various resources interacts and changes over time, our main aim was more restrained. Our interest was in observing how teachers integrated the designed task-sequences into their usual teaching practice, which additional resources they might call upon (e.g., the blackboard and the classroom view-screen – a device connected to an overhead projector that projects the screen display of the calculator that is hooked up to it), and in which ways they might adapt the task-sequence materials. In addition to the videotaped observations of each classroom lesson involving our task-sequences, and the follow-up conversations with each teacher, we also observed a couple of lessons of each teacher's regular teaching practice prior to the start of the research. In sum, the five-month period that we spent in each teacher's class during the main project allowed us not only to gather evidence of each teacher's observable actions with the task-sequences, but also to develop an awareness of some of the underlying factors at play. It is noted that one of the teacher's classes was observed again the following year, during those months in which he was using once more the researcher-designed tasks. Just as has been pointed out by Ruthven (2007), the design elaborated by the researcher-designers was one that continued in usage, but the analysis of those evolutionary changes with respect to the role of the teacher as designer of his own resources is not part of the present chapter.

10.3 The researcher-designed resources

The student version of each task-sequence consisted of a set of activity sheets that presented the task questions and blocked-off spaces for written answers, as well as indications as to when classroom discussion could be expected to occur. The research team also created a teacher/researcher guide to accompany each task-sequence, in addition to a solution key (see the research team's website for the task-sequences: http://www.math.uqam.ca/apte/indexA.html). Thus, while the student task-sequences constitute a central component of the researcher-designed resources, the resources are also understood to include the accompanying teacher guides, the particular CAS tool that was used (along with its guide), and the discussions that were held during the workshop sessions regarding the spirit embedded within the textual materials, as well as any ad hoc conversations that occurred during the unfolding of the research.

10.3.1 Three key features of the researcher-designed task-sequences

In designing the task-sequences, our intentions revolved around three key aspects: the mathematics, the students, and the technology.

Mathematics-wise, all of the task-sequences involved a dialectic between technique and theory within a predominantly exploratory approach, with many open-ended questions. The mathematics that formed the content of the tasks intersected with, but also extended, the usual fare for Grade 10 algebra students. At times, the main mathematical theme of the task-sequence was more technique oriented, as in: factoring the $x^n - 1$ family of polynomials for integral values of *n*, solving systems of linear equations, using factoring to solve equations containing radicals, and exploring the sum and difference of cubes. At other times, the focus was more theoretical in nature, as in the task-sequences related to the equivalence of algebraic expressions. But in both cases, a combination of technical and theoretical activity related to the mathematics was envisaged. In brief, the intended emphases related to the mathematics included: i) coordinating the technical and theoretical aspects of the mathematics, ii) pattern seeking, inductive reasoning, and development of techniques, iii) conjecture making and testing, and iv) deductive reasoning and proof.

Student-wise, we built into the task-sequences not only questions where the students would be encouraged to reflect on their mathematics, but also indicated moments where they would be expected to talk about their mathematical thinking during whole-class discussions. Tasks that asked students to write about how they were interpreting their mathematical work and the answers produced by the CAS aimed at bringing mathematical notions to the surface, making them objects of explicit reflection and discourse in the classroom. In sum, the intended emphases that related to the students included: i) encouraging them to be reflective and inquiring into their thinking, and ii) encouraging them to share their ideas, questions, and conjectures during collective discussions.

Technology-wise, all of the task-sequences involved technical activity with either the CAS, with paper and pencil, or with both. We viewed the CAS as a mathematical tool that, through the task, stimulates reflection and generates results that are to be coordinated with paper-and-pencil work. The CAS served thus as a confirmation-verification tool and/or a surprise generator (producing results that would, in general, not be expected by the students). Very few CAS commands were required for the task-sequences we designed, simply Factor, Expand, Solve, and the evaluation command; thus, the manipulation of the technological tool itself was not to impede the mathematical thinking encouraged by the task-sequences. Additional technologies that we considered would be used included the view-screen and the blackboard. In sum, the intended emphases that related to the technologies included: i) taking advantage of the potential of CAS for producing surprising responses that would provoke a rethinking of techniques or theories, for verifying conjectures of a technical or theoretical nature, and for checking paper-and-pencil work; ii) using the blackboard for rendering public, within class discussions, both teacher explanations and student work.

10.3.2 The issue of explicit versus implicit researcher-designer intentions

The teacher guides, which also contained all of the task questions that were addressed to the students, included many specifics that were addressed to the teacher alone. Firstly, they offered explicit suggestions as to the precise mathematical content that might be addressed within the collective discussions. Secondly, they presented a few examples that illustrated, pedagogically-speaking, how a particular topic might be further explained at the blackboard. The following text from the teacher guide for the first part of the Activity 7 task-sequence illustrates how the researchers made explicit the main mathematical issues for discussion, potential erroneous thinking on the part of the students, and the role of the CAS for the given task questions (see Figure 1).

For discussion:
In the course textbook, taking out a common factor is approached without a clear motivation or rationale for its use. Here, the aim of taking out the common factor $y-2$ is relatively easy to motivate, be it in the expression $(y-2)^3-10(y-2)$ or the expression $(y-2)^3-10(y-2)-y(y-2)$.
In each case, taking out the common factor enables students to reduce the problem to one of solving a quadratic equation (having solutions: $y = 6$ and $y = -1$), whether it be by factoring out $y-2$ on both sides of the equation $(y-2)\left((y-2)^2-10\right)=y(y-2)$, or by invoking the zero-product theorem in the equation $(y-2)\left((y-2)^2-10-y\right)=0$. Moreover, the aim is to orient students to the possible "taking out of the common factor" involving the radical expression in the two subsequent items.

Among those students who take out the common factor $y-2$ on both sides of the equation, some are likely to "lose" the solution $y = 2$. Whether or not this be the case, however, on the basis of this example the teacher should conduct a classroom discussion about what precautions to take before canceling a factor common to both sides of an equation. In effect, for the values of a variable for which the common factor vanishes, this simplification is tantamount to division by zero! Those values of the variable must therefore always be treated (i.e., verified as possible solutions) one by one, before simplification. It is this very simplification, for which the solution $y = 2$, given by the calculator, is lost, that we hope students will retain.

The teacher can also help students see how to avoid this problem by using the strategy consisting of bringing all terms to one side of the equation:

$$(y-2)^3-10(y-2)-y(y-2)=0$$

and invoking the theorem: "a product of two factors is zero iff either one of the factors is zero."

Figure 1. *Intentions of the researcher-designers that were rendered explicit within the teacher guide for Activity 7*

But, in general, the teacher guides did not elaborate on the student-related or technology-related intentions of the researcher-designers. For example, the teacher guides did not specify how to conduct the collective discussions – how to encourage reflection, how to inquire into student thinking, how to have students share their thinking with their classmates during the collective sessions, how to use the blackboard to help students coordinate their CAS and paper-and-pencil techniques, or how to orchestrate discussions of a theoretical nature.

The explicitness of the students' written task-questions was intended, in a sense, to help fill in some of the gaps regarding that which was not communicated explicitly to teachers. The written questions that were directed to the students, and the frequent pointers to whole class discussions, were intended to convey to the teachers, albeit in an implicit way, the researcher-designers' intentions regarding the mathematics, the students, and the technology. For example, task questions such as, "*Explain why* $(x+1)$ *is always a factor of* $x^n - 1$ *for even values of n* ≥ 2", and "*Perform the indicated operation (using paper and pencil):* $(x-1)(x^2+x+1) =$ " were quite specific in their stress on the use of either theoretical or technical means for approaching the mathematics. Similarly, questions that related to mathematical reflection, such as, "*Based on your observations with regard to the results in the table above, what do you conjecture would happen if you extended the table to include other values of x?*", as well as the mention in the Activity sheets that collective discussions were scheduled to follow, were meant to communicate not just the need for student reflection, but also the intention to have students discuss their reflections during the collective sessions. Our technological intentions, especially those regarding the coordinating of paper-and-pencil and CAS techniques, were also explicitly presented in the task questions, for example: "*Verify the anticipated result above using paper and pencil and then using the calculator,*" and "*If, for a given row, the results in the left (with paper-and-pencil) and middle (with CAS) columns differ, reconcile the two by using algebraic manipulations in the right-hand column*."
Thus, the teacher guides were a blend of the implicit and the explicit. Explicit within the structure of the task-sequences were the mathematical aims, the issues on which students were expected to reflect, and the ways in which the CAS and paper-and-pencil technologies were to be used. Implicit was the fact that all three of these were to be combined and coordinated, as well as a manner for doing so, within the collective discussions. As will be seen in the upcoming section, teachers adapted both that which had been rendered explicit, as well as that which had been suggested implicitly, within the researcher-designed resources.

Before presenting the nature of these adaptations, a few additional remarks are in order with respect to both the implicit and its adaptation. In all reading of text, the reader has a part to play.

This notion is discussed in many theoretical writings, including Otte's (1986) complementarist position on the dialectic between textual structure and human activity, as well as Remillard's (Chapter 6) view that, "the form of a curriculum resource includes, but goes beyond, what is seen." Nevertheless, that which is unseen is not necessarily less present than that which is seen, as argued by Helgesson (2002, p. 34): "What is implicit, and thus unstated, is not necessarily less clear (or obvious) or less direct than what is explicitly stated; in other words, that an assumption is implicit does not mean that it is hidden and hard to find, or realized to be there only after some reflection." Helgesson, who defines *implicit* as that which is implied, understood, or inferable – tacitly contained but not expressed – points out that the tone and style in which the text is written may also say something about what it is intended to communicate. In keeping with Helgesson, we consider as implicit those unwritten and unspoken aspects of the researcher-designed resources that can be inferred from that which was explicitly stated, those aspects that could be said to be in the spirit of that which was communicated directly. Also in line with Helgesson, we would argue that the implicit does not necessarily require any additional reflective interpretation than that which is called upon for the explicit. Thus, adapting that which is implicit should be akin to adapting that which is explicit.

10.4 Teachers' classroom adaptations of the researcher-designed resources

The two task-sequences that are the focus of this chapter are Activities 6 and 7. Activity 6 was related to the factoring of $x^n - 1$, for integral values of n (for a different elaboration of this task, see Mounier & Aldon, 1996, whose work provided the initial inspiration for our task-sequence). Activity 7 dealt with the use of factoring to solve equations with radicals. These task-sequences were selected because the two of them taken together highlight the duality of the adaptations made by our teacher participants: (a) adaptations dealing with more implicit aspects of the design and with unspecified areas of the researcher-designed resources, and (b) adaptations related to changing or reorganizing an explicit aspect of the design. The extracts analyzed from Activity 6 will bear on adaptations made to the more implicit intentions of the researcher-designers, with examples drawn from the practice of T1 and T2, while Activity 7 will focus on adaptations to the explicit with examples from T3's practice.

All teacher activity that was addressed to the entire class has been taken into account. In our analysis, we foreground teaching practice against the backdrop of the three overlapping, interrelated key features of our researcher-designed resources: the mathematics-related, the student-related, and the technology-related, within the secondary frame of the implicit versus the explicit.

10.4.1 Adaptations observed during the unfolding of Activity 6

Our analysis begins with the adaptations made to the implicit, unwritten and unspoken aspects of the researcher-designed resources. Activity 6, which included the Telescoping, Reconciling, and Proving Tasks, aimed at having students discover a general pattern for the factorization of $x^n - 1$ and instilling the idea of middle-term cancellation. By working on the reconciliation between CAS and paper-and-pencil factorizations, students were to develop their own factoring abilities and to conjecture and inductively extract some factoring properties. It was intended that they should explore and reflect on their mathematics, constructing and validating their own factoring techniques, and also share their work and their thinking during the collective classroom discussion. Technologically-speaking, it was expected that the CAS be used as a tool for verifying paper-and-pencil work with *factor* and *expand*, and for testing conjectures. We also expected that the surprise brought by the CAS through some of the verifications would lead students to strive for deepening their factorization techniques, but that this would require some additional elaboration presented at the blackboard.

The Telescoping Task. Let us consider the beginning of the first collective discussion within Activity 6, where T2 conveyed his particular style for dealing with mathematical issues of a technical and theoretical sort (see Figure 2). The context was Question 2d: *How do you explain the fact that the following products* $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, *and* $(x-1)(x^3+x^2+x+1)$ *result in a binomial?*

T2: [while writing at the board; see Figure 3] When you expand this $(x-1)(x+1)$ and add all your terms you get (x^2-1). Agree? And for the other one $(x-1)(x^2+x+1)$ the same

idea, I multiply the –1 throughout, getting $-x^2 - x - 1$, and that is going to give you $x^3 - 1$.
What do you notice about the middle parts?
Ss (several students, all at once): They cancel out.
T2: They cancel out, because the *x* just elevates the degree of everything, and when you bring the -1, all the middle terms will cancel. You are going to have your x^3 because you elevated the degree, but you are going to have your -1 at the end as well, and everything in the middle will cancel out. That is why without doing any algebraic manipulations, if I did $(x-1)(x^3+x^2+x+1)$, I notice that these (x^3+x^2+x+1) are just a decreasing degree of *x*, so without doing any distributing, you figure out what the results would be.

Figure 2. *Extract from the discussion surrounding the Telescoping Task in T2's class*

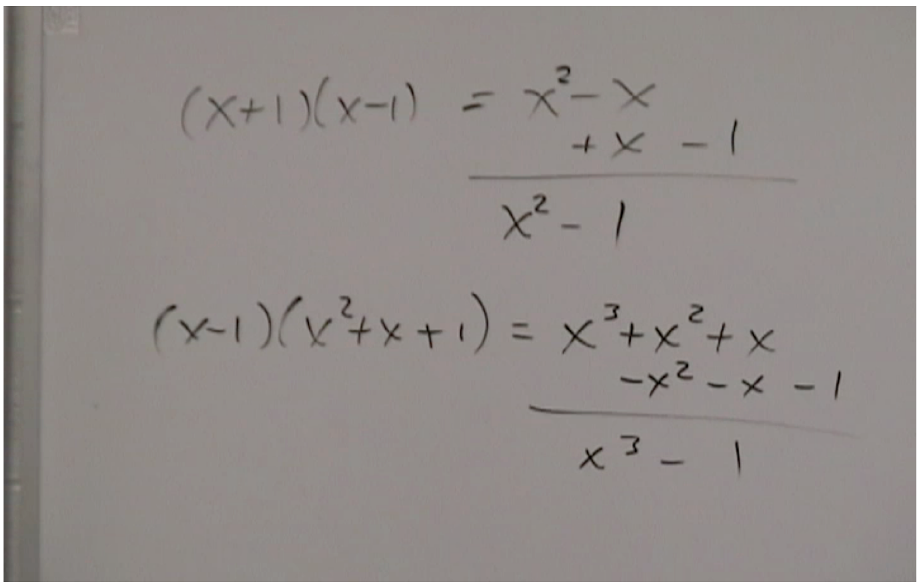

Figure 3. *T2's use of the blackboard during the Telescoping Task*

The technique and the theory of the mathematics are being talked about. But notice that T2 is not drawing these aspects from the students, but is rather presenting them himself. If one could say that our general intention about coordination between technique and theory has not been disregarded, our emphasis on fostering personal mathematical reflection on the part of the students, and on inquiring into their thinking, is clearly set aside by T2's intervention. This is in contrast with T1's style of orchestrating a whole class discussion, as will be seen with the example of the subsequent Reconciling Task, which is provided in Figure 4.

In this activity each line of the table below must be filled in completely (all three cells), one row at a time. Start from the top row (the cells of the three columns) and work your way down. If, for a given row, the results in the left and middle columns differ, reconcile the two by using algebraic manipulations in the right-hand column.

Factorization using paper and pencil	Result produced by the FACTOR command	Calculation to reconcile the two, if necessary
$x^2 - 1 =$		
$x^3 - 1 =$		
$x^4 - 1 =$		
$x^5 - 1 =$		
$x^6 - 1 =$		

Figure 4. *The first part of the Reconciling Task*

The Reconciling Task. For the factoring of $x^4 - 1$, the CAS had not yielded for the students that which they had expected: not $(x-1)(x^3+x^2+x+1)$, but rather $(x-1)(x+1)(x^2+1)$. In T1's class, the following discussion ensued (see Figure 5).

T1: What does it turn out is the case?
S1: Sometimes they like factor even more.
T1: What we did initially is not wrong. It's just not complete. … So for $x^4 - 1$, it's what?
S1: $(x-1)(x+1)(x^2+1)$ [teacher writes at the board: $x^4 - 1 = (x-1)(x+1)(x^2+1)$]
T1: So let's look at this one. How can we go about getting that without the calculator?
S2: Use the rule.
T1: Is that right (as the teacher writes at the board: $(x-1)(x^3+x^2+x+1)$]
Class: Yeah.
T1: And what do you do from there?
S2: Group it.
T1: And how do you group it?
S2: [student explains how she would group the second factor, as the teacher writes at the board that which she dictates]
T1: that's one way of doing it. Bob [S3]?
S3: [the student Bob then describes how he would factor $x^4 - 1$ by first breaking the x^4 part into two equal halves]
T1: What concept have you used?
S3: Difference of squares [the student continues his explanation of the technique, which the teacher writes at the board as per S3's dictation]
T1: So both ways reconcile the differences, coming in from different points of view.

Figure 5. *Extract from the discussion following the Reconciling Task in T1's class*

The extract provided in Figure 5 illustrates the ways in which T1 adapted the researcher-designed resources by filling in some of the unstated gaps in the teacher guide. He inquired into students' thinking and used this as a basis for discussing some of the different approaches to factoring completely $x^4 - 1$. This was done with the stated aim of reconciling the differences between the unexpected result produced by the CAS and the paper-and-pencil result yielded by the general rule. T1 also displayed on the blackboard the various factoring approaches offered by the students, which thereby presented a public record of their different techniques.

This is in contrast to the manner in which T2 responded to the implicit intentions of the researcher-designers for the same task. As seen in the Figure 6 excerpt, T2 used the blackboard to show only the technique that he wanted to emphasize for the factoring of the $x^4 - 1$ binomial: the difference of squares. It would also appear that student participation was called upon with the sole purpose of providing an opening for T2's own preferred approach, the one that he considered more efficient. Other approaches that students offered were dealt with orally.

T2: Now for the $x^4 - 1$, if you use the trick that we were looking at, and we just write it like this [teacher writes on the board $\left(x^4 - 1\right) = \left(x - 1\right)\left(x^3 + x^2 + x + 1\right)$]; this is factored but not fully factored. When you press *Factor* on your calculator, what do you get? What did you get, Chris (S1), when you did *Factor* on your calculator?
S1: $\left(x - 1\right)\left(x + 1\right)\left(x^2 + 1\right)$
T2: [teacher wrote this response on the board] Right! Like that. So, how do we reconcile the two? …
S2: You could do difference of squares at the start.
T2: Yes, you go back to the start, and that is what I said, you can go back to the start, and look at how you do it paper-and-pencil-wise. If you go back to the start and you've got your $\left(x^2 - 1\right)$, your $\left(x^2 + 1\right)$ [teacher writes these two factors on the blackboard] – your difference of squares – and then you have another difference of squares here [teacher points to $\left(x^2 - 1\right)$ and writes $\left(x - 1\right)\left(x + 1\right)$ below it]. … So, in another words, what we are discovering is that our little trick that we did, that only helps to get the $\left(x - 1\right)$ out. That doesn't necessarily mean that what is left is not refactorable. …
S3: I did the opposite, I mean, [$\left(x - 1\right)\left(x + 1\right)\left(x^2 + 1\right) = \left(x - 1\right)\left(x^3 + x^2 + x + 1\right)$]
T2: Reconciling the two doesn't mean just expanding one and showing it is $x^4 - 1$, and I guess that's what you are saying.
S2: Instead, we can just, eh, factor out the other x^2 and make it, for the second factor.

T2: So, you grouped two by two. So, that is another way you could have factored this bracket over here. Because it is four terms, you factor out x^2 here; you get $(x+1)$, then you factor your $(x+1)$ out, and you get $(x+1)$ and (x^2+1). Ok? All right. [teacher does not write the grouping method out on the blackboard; he just points to the different terms and states orally S2's method]

Figure 6. *Extract from the discussion surrounding the Reconciling Task in T2's class*

Another instance of T2's style of adapting what was implicitly conveyed in the resources concerns the factoring of $x^{10}-1$. Here, the surprise factor of the CAS tended not be taken advantage of, nor allowed to play its intended thought-provoking role. While students were still working on the second part of the Reconciling Task, with the polynomials from x^7-1 to $x^{13}-1$, trying to reconcile their paper-and-pencil factorizations with the results produced by the CAS, T2 rapidly wrote $(x^{10}-1)=(x^5-1)(x^5+1)$ on the board. He then stated: "The one that may give you some trouble here is the *x* to the 10*th*. I will explain why." He proceeded to explain at the board the factorization $x^5+1=(x+1)(x^4-x^3+x^2-x+1)$, with a great deal of hand-waving. He then tried to generalize the alternating signs and telescoping argument, but his explanation – an adaptation probably done on the fly – was incomplete and fuzzy, with odd powers that were multiples of three considered separately from other odd powers.

A rather different situation evolved in T1's class where the $x^{10}-1$ example led a student to conjecture a new theory involving the factoring of x^n+1 for odd *n*s – based on the CAS factorization of x^5+1, supported by the factoring pattern for the sum of cubes, x^3+1 (for more on the unfolding of this student's conjecture, see Kieran & Guzmán, 2010). T1 encouraged the student to talk about the way he was thinking and to be as complete as possible in his explanation.

The Proving Task. T1 believed that students would need time to get into the last task of the sequence, the Proving Task: *Explain why* $(x+1)$ *is always a factor of* x^n-1 *for even values of* $n \geq 2$. (Note that the teacher guide had not included any explicit suggestions in regard to the proving task, simply a possible solution based on the Factor Theorem, i.e., "*a* is a zero of the polynomial $p(x)$ iff $(x-a)$ is a factor of $p(x)$".) T1 waited patiently until some of the students had ideas to submit to the class. He then asked three of them to go to the board in turn and to write down and explain their proofs. He requested that the class listen carefully to the explanations being offered by these proof-givers: "Guys, give him a chance" and "Ok, listen because this is interesting, it's a completely different way of looking at it". After each of their explanations, everyone in the class was encouraged to discuss and try to understand the main approach used in the proof. From time to time, T1 asked for further clarification, offered counter-examples, and pushed students to think more deeply. T1's way of filling the gaps in the teacher guide for the Proving Task was in sync with the student-related intentions of the researcher-designers, and even enriched them further.

This contrasts once again with T2, who allocated little time for the students to think about the Proving Task. A couple of students proposed different avenues to pursue, but T2 seemed in such a hurry to give his own proof, based on the Factor Theorem, that he asked them neither to come to the board nor to explain fully their way of thinking. It appeared here and elsewhere in Activity 6 that T2 assumed ownership of all the main mathematical ideas presented in class, a corollary being that students were not held responsible for thoroughly explaining their own thinking.

10.4.2 Adaptations observed during the unfolding of Activity 7

Our analysis continues, this time bearing on the adaptations made to the explicit, written and spoken aspects of the researcher-designed resources. Mathematics-wise, our primary intention in Activity 7 was to make students aware of the possible loss of solutions when they simplify an equation by dividing both sides by some factor. Students were thereby to be directed towards the more reliable solving method of isolating terms on one side and using the zero-product theorem, that is, "a product is zero iff either one of the factors is zero". Both the teacher guide and the student task-sequence had included explicit mathematical notes in the opening block of Activity 7 (see Figure 7).

> *Primary idea*: Factoring (taking out a common factor) as a tool for solving equations, particularly when used in conjunction with the "zero-product theorem".
> *Secondary ideas*:
> - Factoring (taking out a common factor) can be applied not only to constants and variables, but also to algebraic expressions that can be taken as objects to operate upon.

- Students should be able to bring the methods learned for solving linear and quadratic equations to bear on equations that are neither linear nor quadratic, per se;
- Simplifying an equation by dividing both sides by some factor may lead to a loss of solutions. In equations in which such simplifications are possible, the strategy of isolating terms on one side of the equation and using the zero-product theorem is generally a more effective solving method;
- In equations involving variables under the radical sign, verification after solving is not only advisable, but necessary.

Figure 7. *Explicit mathematical notes in both the student task-sequence and the teacher guide*

Student-wise, it was intended, and explicitly asked for, that they describe at a meta-level – perhaps quite loosely – both the patterns they were seeing within the equation and their equation-solving approach, before actually solving the equation. We were expecting that, for the subsequent equation-solving task, most of them would lose a solution. Technology-wise, the CAS was to be used as a follow-up to the paper-and-pencil solving – a verifying device that would yield surprises, such as producing one more solution than they had likely obtained with their paper-and-pencil methods. The teacher's guide suggested a way of handling the class discussion related to lost solutions and their verification with the CAS (see Figure 1, presented earlier in this chapter). In sum, the central explicit components related to the first three tasks of Activity 7 concerned, in this order: (a) a focus on the meta-level aspects of solving a particular equation containing common factors with radicals, (b) the actual solving of a related equation having a similar pattern of common factors (but without radicals) and which could induce a loss of solutions, and (c) the verification by CAS of the paper-and-pencil solutions which would lead for many students to a required reconciliation of the two sets of solutions.

All of the examples given in this section are based on T3's practice, as it was here that we observed the most extensive adaptations to the explicit aspects of the researcher-designed resources. Specifically, his adaptations involved replacing an expression by a letter, inserting a transitional equation, and using the CAS to factor a quadratic. In addition, we note that T3 had an empathetic way of preparing students for possible task difficulties, telling them not to worry and reformulating each question with a phrasing that in his view was better adapted to their level of understanding. A notable sign of his general attitude is the fact that from the outset of the prior

Activity 6, in advance of bringing it to class, he told the researchers that he had decided to skip the final Proving Task, it being too difficult in his view. In his defence, recall that T3's class was the weakest of the three.

Replacing an Expression by a Letter. For the first proposed equation in Activity 7, $5\left(\sqrt{x-4}\right)^3+11\sqrt{x-4}=\left(2x+1\right)\sqrt{x-4}$ (Equation 1), a general reflection on how students would proceed to solve this equation was to be elicited. Immediately afterward, they were to be directed toward the simpler equation $\left(y-2\right)^3-10\left(y-2\right)=y\left(y-2\right)$ (Equation 2), which was the one to be actually solved. From the start, while reading and rewording the instructions, before anything whatsoever had been done by his students, T3 suggested replacing $\sqrt{x-4}$ by *a* (see Figure 8).

> T3: Just imagine that this expression, root of *x* minus 4, is replaced by *a*. It would give you something much simpler, wouldn't it? So each time it's written 'root of *x* minus 4', if we write *a* instead, we would have quite a simpler equation and then, what would you do? Because the principle would be, say, quite the same. Anyway, some aspects would be the same. So try to see what you could do to solve this equation ... or anyway, what would be your steps. We don't ask you to solve it, just go with what you think you would do. There are no bad answers, just try.

Figure 8. *T3 suggested to students that they replace an expression by a letter*

This adaptation interfered with our intention of having students recognize by themselves in what facet Equations 1 and 2 have the same structure, and to what extent the solving steps they were asked to sketch for Equation 1 could be put to the test by actually solving Equation 2. As well, we note that T3 did not follow the explicitly-given sequence of holding off on the class discussion until after the students had worked on both Equations 1 and 2 and had tested the solutions of Equation 2 with the CAS. Following his too early and wordy discourse on Equation 1, T3 had students work on Equation 1, but in fact never asked them how they viewed it at a meta level.

Inserting a Transitional Equation. We will now see that T3's implementation of the activity digressed even further from that which was explicitly presented in the researcher-designed resources (see Figure 9).

T3 [reading]: "Using paper and pencil, see whether you can first solve the following equation". So they are giving you another one that may be less scary, which is *y* minus 2 to the three... [He does not finish reading the second equation, but searches for a piece of chalk]. Well... They are giving you this one so that you can compare, they say that it is somewhat analogous to the 'monster' given just before. They say [reading]: "Factoring (taking out a common factor) might be useful here". So they are giving you a hint. Ok, I have seen that some of you wrote interesting things, in the sense that you have already good ideas about how to solve. Nevertheless I'll give one to you all, because some of you are facing it without knowing what to do, and that I can understand.
I'll do an example that is completely different.
[He writes on the board: $5(a-3)+2(a-3)^2=3(a-3)^3$.] I'm coming back to what I've said before, about root of *x* minus 4, if I remember well, that it could be replaced by a value, say, *a*. Ok, it may have given some of you a hint, precisely about what could be done with it. So here [he gets a piece of colored chalk and circles $(a-3)$], if I say here that *a* minus 3, if all of this parenthesis here would have been, say, *x*. We would be facing [he writes on the board: $5x+2x^2=3x^3$]. Do you agree? All I have done is that I've been saying to myself: Instead of writing $a-3$, to make things easier, I'll replace $a-3$ by *x*. And then, I'm facing this new equation. Is this equation [pointing to the board] less scary?

Figure 9. *T3's use of a transitional equation*

T3's insertion of a transitional equation, accompanied by replacing the main expression by a letter, was an adaptation that not only further confounded our initial intentions with respect to students' seeing structural similarities between the two equations, but also presented an added mathematical difficulty for the students: Equation 2 conveying a term in both y and $y-2$, the substitution of x for $y-2$ gives either a two-variable equation or a term in x and $x+2$. The transitional equation introduced by T3 did not involve such a hindrance. Whether T3 proposed it as a transitional stage for the students, or simply did not foresee this snag pertaining to substitution in Equation 2, we do not know. In any case, as the students began working on Equation 2, one did complain that the substitution of x for $y-2$ gave him an xy term, which got him stuck. T3 offered him the following hint: "Nothing keeps you from going back to $y-2$." Still a little later, as T3 was showing at the board a method for handling Equation 2, he replaced the $y-2$ by a (while keeping a term in ay), factoring out an a (see Figure 10) and replacing back the a by $y-2$. (The possibility of substituting $a+2$ for y was not mentioned.) Finally, the equations displayed on the blackboard by T3, as a path to solve Equation 2, were:

$a\left(a^2-10-y\right)=0$, $\left(y-2\right)\left(\left(y-2\right)^2-10-y\right)=0$, and $\left(y-2\right)\left(y-6\right)\left(y+1\right)=0$. The fact that T3 left out many of the intermediate steps of the solving process and used oral commentary to fill the missing steps is quite surprising, in view of his meta-level remarks to the students on how difficult they must be finding this work.

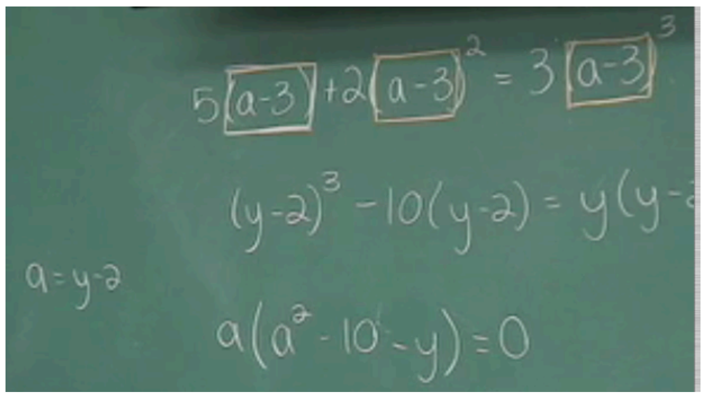

Figure 10. *An unexpected transitional equation before solving Equation 2*

Using CAS to factor a quadratic. Further adaptations by T3 concerned his use of the CAS technology. When discussing the solving of $-3x^3+2x^2+5x=0$ (derived from the transitional equation $5\left(a-3\right)+2\left(a-3\right)^2=3\left(a-3\right)^3$ by substituting x for $a-3$), T3 suggested that, the common factor x having been taken out, students may then use the CAS to factor $-3x^2+2x+5$. Perhaps this suggestion was made in the interests of time or to reduce some of the overall complexity of the task. Nevertheless, it was outside the suggested route of solving with paper-and-pencil and, only later, verifying the solutions with the CAS. Moreover, for the third question of this first part of the task-sequence – the one that asked students to check their solutions of Equation 2 with those produced by the CAS – T3 chose to eliminate this question, having introduced Equation 2 with a view-screen display of the three solutions yielded by the CAS and subsequently asking students to find themselves the same three solutions with paper and pencil. Thus, the surprise realization that there might be three solutions, and how it came to be that one of them had been lost through their paper-and-pencil techniques, was never provoked in T3's class.

10.5 Discussion

10.5.1 The implicit versus the explicit in task-designers' intentions

Our analysis of the ways in which teachers adapted the researchers' *implicit* intentions with respect to Activity 6 indicated adaptive activity not only in all three of the key features of the task-based resources but also in their coordination. We observed that our implicit emphasis on fostering personal mathematical reflection on the part of the students, and on inquiring into their thinking, was set aside by most of T2's interventions. Similarly, he used the blackboard to show only the techniques that he wanted to stress. In a related manner, the surprise factor of the CAS was not taken full advantage of with respect to provoking student thinking. In contrast, our analysis of T1's adaptive activity indicated a different manner of filling in the unstated gaps in the teacher guide. He thoroughly inquired into students' thinking and used this as a basis for class discussions. T1 also displayed on the blackboard the various approaches offered by the students, thereby presenting a public trace of their different techniques. Our analysis of the ways in which T3 adapted the researchers' *explicit* intentions with respect to Activity 7 disclosed significant adaptive activity with respect to that which was explicitly documented in the resources regarding the mathematics, the students, and the technology.

Researchers (e.g., Freeman & Porter, 1989) have argued that, if teachers' guides were more explicit and less ambiguous, the degree of closeness between teaching practice with these resources and the intentions of the resource designers could be greater. For example, Manouchehri and Goodman (1998) have critiqued certain reform-based curricula for not "providing the teachers with detailed methods of how to address the content development" (p. 36). Our findings are in disagreement with the argument that greater detail will necessarily lead to a closer following of curriculum materials. No matter how explicitly expressed the researcher-designers' intentions be, adaptation of the resources will take place. Our comparison of the nature of the adaptations that were forged with respect to both the implicitly-suggested and explicitly-expressed intentions of the researcher-designers showed that, in both intentional domains, teachers will adapt the resources that they use. This is not to suggest that researchers and curriculum developers should not attempt to make as explicit as possible their intentions with respect to the content and use of the resources they design, but that they should not expect fidelity. They should rather expect that – as will be discussed shortly – the personal beliefs, goals, and habitual classroom practice of the teachers may be at variance with the epistemological and pedagogical assumptions underlying the researcher-designed resources and that this will inevitably lead to adaptation.

10.5.2 The underpinnings of the adaptations that teachers made to the resources

Our finding that, whether the intentions of the researcher-designers were explicitly stated or implicitly suggested, teachers adapted the given resources leads naturally to the question as to what it was that underpinned these adaptations, that is, why were the resources adapted in the ways that they were? An additional question concerns the issue of the consistency of these adaptations within individual teachers.

The teachers' mathematical knowledge clearly filtered their interpretation of the mathematical intentions of the researcher-designers. This was seen in T3's inappropriate choice of example that he used when inserting his *substitution* technique into the task-sequence. Pedagogical content knowledge also played a role. It underpinned the well-developed ways in which T1 orchestrated the whole-class discussions with students being asked to explain their thinking to the class at large, the differential ways in which T1 and T2 used the blackboard for keeping a written trace of students' thinking and as a tool for mediating the reconciliation of paper-and-pencil and CAS responses, and the varying roles for the CAS technology that were encouraged by each of T1, T2, and T3.

Teachers' beliefs accounted for much of their adaptive activity with respect to their use of the researcher-designed resources, from T1 who believed his students could and should be challenged mathematically and thereby adapted the unfolding of the task-sequences in such a way that students be held responsible for their own mathematical thinking, to T2 who believed that the teacher is the mathematical focal point of the classroom, to T3 who believed his students required a certain social and mathematical security net.

In keeping with Schoenfeld (1998), we have attributed beliefs and knowledge to be at the root of the adaptations that the individual teachers carried out in their day-to-day teaching with our resources. However, such attributions do not account entirely for the global picture of each teacher's approach to adaptation. The enactment of a teacher's beliefs, which translates into both short- and long-term goals in the classroom, also constitutes a 'pedagogical contract' with the students – a certain set of expectations of the teacher for the students and vice versa (note that this pedagogical contract takes in the 'didactical contract' of Brousseau, 1997, but also includes more, namely certain attitudes, beliefs, and convictions of the teacher that are not tied specifically to the mathematical content being considered).

For example, T3 was extremely sensitive to the perceived needs and abilities of his students, as inferred from his rewording of the task questions and alteration of the content so as to try to make the mathematics more accessible to them. He seemed reluctant to put his students into potentially awkward situations where they might not know how to express themselves; thus he engaged the class in very few collective discussions. He called upon students whom he thought might have the beginnings of an answer and then proceeded to elaborate on their rather sketchy responses. In short, he made few mathematical demands of his students. His interactions with the students always weighed on the side of showing empathy toward them.

In contrast, T2 delighted in demonstrating his mathematical prowess to the students. This seemed to be a central part of the identity he had forged for himself in the mathematics classroom. His students, who were very bright, also seemed to appreciate his shows of mathematical bravado. He used the beginnings of students' oral answers as the spark for his own elaborations of the underlying mathematics. He never asked students to respond more fully, but rather attempted to anticipate the direction in which their thinking was headed. There seemed to be an unwritten contract between him and his students that he was the main mathematical resource of the classroom.

A rather different pedagogical contract was at play in T1's class. T1, who was highly respectful of his students, not only encouraged the expression of their mathematical thinking but also asked them for further explanation and justification. During the whole-class discussions, he often assumed what we came to call his *discussion posture* – sitting on the edge of one of the empty student-desks at a front corner of the classroom, thereby indicating to the class that it was now time for some serious collective thinking and sharing of ideas. He intended that students be pushed mathematically and had confidence that they could rise to the occasion, if encouraged to do so – which they did.

The three teachers' deeply held beliefs, which constituted a manner of interacting with their students, lent a certain consistency to their individual adaptations of our resources. This consistency was also seen when impromptu activity occurred. For example, when faced with the unexpected proofs generated by a few of the students, T1 asked the students to come to the front to explain their thinking to the rest of the class; these exposés were then followed by classroom discussion of the central ideas of the proofs. In contrast, T2 when similarly faced with an unexpected proof idea from a student tried to interpret it on his own and illustrate it himself at the board. T3, as per his intentional goals vis-à-vis his students, decided not to embark at all on the proving task. Thus, even if the unexpected led to different ways of handling the situation, each teacher acted on the spur of the moment in ways that were consistent with his own convictions and ways of interacting with his class. This finding makes contact with Remillard's (Chapter 6) observation that teachers' *modes of engagement* with resources are shaped by their expectations, beliefs, and routines, thereby bestowing a degree of stability on these modes. Sensevy (Chapter 3) makes a similar point with regard to the enactment of the threefold process of teaching practice by which documents, prior intentions, and intentions *in action* are intimately linked together. Also related to this discussion are the findings reported by Drijvers (Chapter 14), who describes the unfolding of teachers' intentions with respect to their classroom use of computer-based resources in terms of *didactical configuration*, *exploitation mode*, and *didactical performance*.

Lagrange and Monaghan (2010) have argued that inconsistency characterizes the practice of teachers in dealing with the complexity of classroom use of technology. While we would agree that the presence of the CAS technology within our researcher-designed resources led to more unplanned and impromptu activity than might otherwise be the case in a mathematics class, we would have to disagree with the substance of their claim. We argue instead that the manner in which individual teachers engaged in this impromptu activity was indeed consistent. An example involves the unexpected complete factorization of $x^{10}-1$ by the CAS with its unanticipated factorization of the $\left(x^5+1\right)$ factor. This led to on-the-fly decision-making on the part of both T1 and T2: for T2, it was to gain control of the mathematical situation by having himself present to the class the factorization of this ‘new’ class of expressions; for T1, it was to give the student who was provoked into thinking about a new factorization rule for x^n+1 the time to express his new conjecture and the examples that were supporting it. Just as with the proof example above, T1 and T2 each handled differently the impromptu foray occasioned by unexpected results with the technology; nevertheless, their approaches were clearly consistent with their individual deep-seated beliefs and habitual manner of interacting with their students.

10.6 Concluding remarks

Our study found that adaptive shaping occurred with respect to all three key features of the researcher-designed resources (the mathematics, the students, and the technology), whether our intentions with respect to those features were explicitly stated or implicitly suggested. The results of this study have also shown how embarking with the same set of task-sequences, and sharing the same goal of participating in a research project aimed at developing the technical and theoretical knowledge of algebra students within a CAS-supported environment, can lead to quite different uses of the resources. The teachers brought into the study their own beliefs, knowledge, and customary ways of interacting with their students. Quite clearly this had an impact for each class on the nature of the mathematical activity engaged in. Different adaptations of the same resources either promoted or impeded the emergence of different techniques and theoretical-conceptual elements. But that is a whole other story.

Acknowledgments

We express our appreciation to André Boileau, who contributed to the discussions leading to the writing of this chapter, to the three teachers featured herein, and also to those who collaborated in designing the task-sequences: André Boileau, Fernando Hitt, José Guzmán, and Luis Saldanha, as well as to Michèle Artigue, Paul Drijvers, and Luc Trouche for their thought-provoking contributions to the research. The support of the Social Sciences and Humanities Research Council of Canada (Grant # 410-2007-1485) is gratefully acknowledged. We also thank the editors and reviewers for their helpful feedback on an earlier version of this chapter.

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