TEACHERS PARTICIPATING IN A RESEARCH PROJECT ON LEARNING: THE SPONTANEOUS SHAPING OF RESEARCHER-DESIGNED RESOURCES WITHIN CLASSROOM TEACHING PRACTICE

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Teachers participating in a research project on the learning of algebra with CAS technology spontaneously adapted the resources designed specifically by the researchers for the project. Analysis of the classroom-based observations of teaching practice showed that adaptive shaping occurred with respect to all three key features (the mathematics, the students, and the technology) of our researcher-designed resources, whether our intentions with respect to those features were explicitly stated or implicitly suggested. Using the framework of the "documentational approach of didactics," the results highlight the differential role that the same resources can play vis-à-vis the dialectical processes of 'documentational genesis' whereby resources are viewed as both shaping and being shaped by individual teaching practice.

INTRODUCTION AND BRIEF LITERATURE REVIEW

Mathematics education research has, over the years, yielded numerous resources. But little is known about the ways in which teachers take on such research-based resources and adapt them to their own needs. In 2000, Adler proposed that, "mathematics teacher education needs to focus more attention on resources, on what they are and how they work as an extension of the teacher in school mathematics practice" (Adler, 2000, p. 205). However, much of the resource-related research has focused more on the mathematical design of the resources (e.g., Ainley & Pratt, 2005) or on their impact with respect to student learning (e.g., Hershkowitz et al., 2002), rather than on the ways in which the resources are used by teachers.

The manner in which teachers adapt researcher-designed resources – as opposed to commercially-based resources – is a new area of research, but one that fits into the recently emerging frame of the *documentational approach of didactics* (Gueudet & Trouche, 2009, in press). Within this frame, documents are considered central to didactic phenomena – but so too is the teacher, as indicated by the pivotal construct of *documentational genesis* with its dialectical processes involving both the teacher's shaping of the resource and her practice being shaped by it. Building on a distinction introduced by Rabardel (1995), Gueudet and Trouche (in press) emphasize that not only does the teacher guide the way the resource is used, but also that the affordances and constraints of the resource influence the teacher's activity. As they point out, "design and enacting are intertwined." However, within this framework, little research has of yet used the design characteristics of given resources as a focal lens

2011. In xxxx (Eds.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. XXX-YYY. Ankara, Turkey: PME.

for studying teachers' enactive shaping of them. Remillard (2005), in her review of the research literature on teachers' use of mathematical curricula, argues that features of the curriculum matter to curriculum use as much as characteristics of the teacher and that such research is rather unexplored terrain. In the spirit of Remillard, this report uses the main features of the researcher-designed resources as a backdrop for analyzing their adaptation by participating teachers within their teaching practice.

METHODOLOGICAL CONSIDERATIONS

Participants

When our research team began to create task-sequences for the technical and theoretical development of algebra learners, we also contacted several mathematics teachers. In a workshop setting, the volunteer teachers gave us feedback regarding the nature of the tasks that we were conceptualizing. The week-long workshop included discussions regarding the main mathematics-related and technology-related intentions of the researcher-designers. After modifying the task-sequences in the light of the teachers' feedback, we observed their integration of the task-sequences into their regular mathematics teaching of Grade 10 classes over a five-month period.

The three teachers who are featured in this report all taught in the same city and thus shared a common curricular experience. They are named T1, T2, and T3 (and we use the masculine gender to refer to each of them). T1, whose undergraduate degree was in economics, had been teaching mathematics for five years. He considered his class of students to be of medium-high mathematical ability. T2, who was the most mathematically qualified of the three teachers, had previously taught college-level mathematics during 8 years, before teaching at the secondary school for another 8 years. T2's students were in the top mathematics class. T3, whose undergraduate degree was in the teaching of high school mathematics, had five years of experience at the secondary level. T3's students were weaker in algebra than the other students.

Three key features of the researcher-designed resources

This study is part of a larger program of research whose first phase was oriented toward student learning (see, e.g., Kieran & Drijvers, 2006). The second phase, oriented toward teaching practice, included secondary analyses of the first-phase video-data; these provide the foundation for this report. The analysis centres on teachers' adaptations to the three key features of the researcher-designed resources – the mathematics, the students, and the technology – with a particular focus on whether the adaptations related to an explicit or an implicit aspect of the resources. The student versions of the task-sequences, presented in the form of Activity packets, constitute a central component of the researcher-designed resources; however, the resources also include the accompanying teacher guides, the particular CAS tool that was used (along with its guide), and the discussions that were held during the workshop sessions regarding the spirit embedded within the textual materials, as well as any ad hoc conversations that occurred with the teachers after each of their lessons.

Mathematics-wise, all of the task-sequences involved a dialectic between technique and theory within a predominantly exploratory approach, with many open-ended questions. In brief, the intended mathematical emphases included: i) coordinating the technical and theoretical aspects, ii) pattern seeking, inductive and deductive reasoning, and developing technique, and iii) conjecture making, testing, and proving. Student-wise, we built into the task-sequences questions where the students would be encouraged to reflect on their mathematics, and also indicated moments where they would be expected to talk about their mathematical thinking during whole-class discussions. Technology-wise, all of the task-sequences involved technical activity with either the CAS, with paper and pencil, or with both. We viewed the CAS as a mathematical tool that, through the task, stimulates reflection and generates results that are to be coordinated with paper-and-pencil work. The CAS served thus as a confirmation-verification tool and/or a surprise generator (producing results that would likely not be expected by the students). Additional expected technologies included the CAS view-screen and the blackboard.

The issue of explicit versus implicit researcher-designer intentions

The teacher guides included many specifics that were directed to the teacher alone. Firstly, they offered explicit suggestions as to the precise mathematical content that might be addressed within the collective discussions. Secondly, they presented a few examples that illustrated, pedagogically-speaking, how a particular topic might be further explained at the blackboard. But, in general, the teacher guides did not elaborate on the student-related or technology-related intentions of the researcher-designers. For example, the teacher guides did not specify how to conduct the collective discussions – how to encourage reflection, how to inquire into student thinking, how to have students share their thinking with their classmates during the collective sessions, how to use the blackboard to help students coordinate their CAS and paper-and-pencil techniques, or how to orchestrate theoretical discussions.

The specificity of the students' written task-questions was intended, in a sense, to help fill in some of the gaps regarding that which was not communicated explicitly. Thus, the teacher guides, which included a copy of the student task-questions, were a blend of the implicit and the explicit. Explicit within the structure of the task-sequences were the mathematical aims, the issues on which students were expected to reflect, and the ways in which the CAS and paper-and-pencil technologies were to be used. Implicit was the fact that all three of these were to be combined and coordinated, as well as a manner for doing so, within the collective discussions.

A few theoretical remarks regarding both the implicit and its adaptation are in order. In all reading of text, the reader has a part to play. This notion is discussed in many theoretical writings, including Otte's (1986) complementarist position on the dialectic between textual structure and human activity, as well as Remillard's (in press) view that "the form of a curriculum resource includes, but goes beyond, what is seen." Nevertheless, as argued by Helgesson (2002, p. 34): "What is implicit, and thus unstated, is not necessarily less clear (or obvious) or less direct than what is explicitly stated; in other words, that an assumption is implicit does not mean that it is hidden and hard to find, or realized to be there only after some reflection." Helgesson, who defines *implicit* as that which is implied, understood, or inferable – tacitly contained but not expressed – points out that the tone and style in which the text is written may also say something about what it is intended to communicate. In keeping with Helgesson, we consider as implicit those unwritten and unspoken aspects of the researcher-designed resources that can be inferred from what was explicitly stated, those aspects that could be said to be in the spirit of what was communicated directly. Also, like Helgesson, we would argue that the implicit does not necessarily require any additional reflective interpretation than that which is called upon for the explicit.

TEACHERS' CLASSROOM ADAPTATIONS OF THE RESOURCES

The two task-sequences that are the focus of this report are Activities 6 and 7 (for the full set of task-sequences: http://www.math.uqam.ca/apte/indexA.html). Activity 6 was related to the factoring of $x^n - 1$, for integral values of n (a task-sequence inspired by Mounier & Aldon, 1996). Activity 7 dealt with the use of factoring to solve equations with radicals. The extracts analyzed from Activity 6 will bear on adaptations made to the *implicit* aspects of the design, with examples drawn from the practice of T1 and T2. Activity 7 will focus on adaptations related to changing or reorganizing an *explicit* aspect of the design, with examples from T3's practice.

Examples of Adaptations Observed During the Unfolding of Activity 6

This analysis begins with the adaptations made to the implicit, unwritten and unspoken, aspects of the researcher-designed resources. For our first of two examples drawn from Activity 6, we examine the beginning of the first collective discussion within the activity, where T2 conveyed his particular style for dealing with mathematical issues of a technical and theoretical sort (see Figure 1). The context was Question 2d: *How do you explain the fact that the following products* $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, *and* $(x-1)(x^3+x^2+x+1)$ *result in a binomial?*

T2: [while writing at the board] When you expand this $(x-1)(x+1)$ and add all your terms you get (x^2-1). Agree? And for the other one $(x-1)(x^2+x+1)$ the same idea, I multiply the -1 throughout, getting $-x^2-x-1$, and that is going to give you x^3-1. What do you notice about the middle parts?
Ss (several students, all at once): They cancel out.
T2: They cancel out, because the x just elevates the degree of everything, and when you bring the -1, all the middle terms will cancel. You are going to have your x^3 because you elevated the degree, but you are going to have your -1 at the end as well, and everything in the middle will cancel out. That is why without doing any algebraic manipulations, if I did $(x-1)(x^3+x^2+x+1)$, I notice that these (x^3+x^2+x+1) are just a decreasing degree of x, so without doing any distributing, you figure out the results.

Figure 1. *Extract from discussion surrounding Question 2d in T2's class*

The technique and the theory of the mathematics are being talked about. But notice that T2 is not drawing these aspects from the students, but is rather presenting them himself. If one could say that our general intention about coordination between technique and theory has not been disregarded, our implicit intention with respect to fostering personal mathematical reflection in students, and on inquiring into their thinking, is clearly set aside by T2's intervention. This is in contrast with T1's style of orchestrating a whole-class discussion, as is seen with the next example involving elements from the subsequent task of Activity 6.

For the factoring of $x^4 - 1$, the CAS had not yielded what the students had expected: not $(x-1)(x^3+x^2+x+1)$, but rather $(x-1)(x+1)(x^2+1)$. In T1's class, the following discussion ensued (see Figure 2).

T1: What does it turn out is the case?
S1: Sometimes they like factor even more.
T1: What we did initially is not wrong. It's just not complete. … So for $x^4 - 1$, it's what?
S1: $(x-1)(x+1)(x^2+1)$ [teacher writes at the board: $x^4 - 1 = (x-1)(x+1)(x^2+1)$]
T1: So let's look at this one. How can we go about getting that without the calculator?
S2: Use the rule.
T1: Is that right (as the teacher writes at the board: $(x-1)(x^3+x^2+x+1)$]
Class: Yeah.
T1: And what do you do from there?
S2: Group it.
T1: And how do you group it?
S2: [student explains how she would group the second factor, as the teacher writes at the board that which she dictates]
T1: That's one way of doing it. Bob [S3]?
S3: [the student Bob then describes how he would factor $x^4 - 1$ by first breaking the x^4 part into two equal halves]
T1: What concept have you used?
S3: Difference of squares [the student continues his explanation of the technique, which the teacher writes at the board as per S3's dictation]
T1: So both ways reconcile the differences, coming in from different points of view.

Figure 2. *Extract from the discussion on the factoring of* $x^4 - 1$ *in T1's class*

The extract provided in Figure 2 illustrates the ways in which T1 adapted the researcher-designed resources by filling in some of the unstated gaps in the teacher guide. He inquired into students' thinking and used this as a basis for discussing some of the different approaches to factoring completely $x^4 - 1$. This was done with the stated aim of reconciling the differences between the unexpected result produced by the CAS and the paper-and-pencil result yielded by the general rule. T1 also displayed on the blackboard the various factoring approaches offered by the students, which thereby presented a public record of their different techniques.

Examples of Adaptations Observed During the Unfolding of Activity 7

Our analysis continues, this time bearing on the adaptations made to explicitly-stated aspects of the researcher-designed resources, with examples drawn from the practice of T3. Mathematics-wise, our primary intention in Activity 7 was to make students aware of the possible loss of solutions when they simplify an equation by dividing both sides by some factor. Students were thereby to be directed towards the more reliable solving method of isolating terms on one side and using the zero-product theorem, that is, "a product is zero iff either one of the factors is zero". The teacher's guide suggested a way of handling the class discussion related to lost solutions and their verification with the CAS. In brief, the central explicit components related to the first three tasks of Activity 7 concerned, in this order: (a) a focus on the meta-level aspects of solving a particular equation containing common factors with radicals, (b) the actual solving of a related equation having a similar pattern of common factors (but without radicals) and which could induce a loss of solutions, and (c) the verification by CAS of the paper-and-pencil solutions which would lead for many students to a required reconciliation of the two sets of solutions.

T3 carried out several adaptations to these explicit aspects of the task-sequence. For Equation 1, $5\left(\sqrt{x-4}\right)^3 + 11\sqrt{x-4} = \left(2x+1\right)\sqrt{x-4}$, a meta-level reflection question asked students how they would approach the solving of this equation. Immediately afterward, the written task-sequence directed them toward the simpler equation $\left(y-2\right)^3 - 10\left(y-2\right) = y\left(y-2\right)$ (Equation 2), which was the one to be actually solved. But from the start, while reading aloud and rewording the instructions, before anything had been done by his students, T3 suggested replacing $\sqrt{x-4}$ by a.

This adaptation interfered with our intention of having students recognize by themselves in what facet Equations 1 and 2 have the same structure, and to what extent the solving steps they were asked to sketch for Equation 1 could be put to the test by actually solving Equation 2. As well, we note that T3 did not follow the explicitly-given sequence of holding off on the class discussion until after the students had worked on both Equations 1 and 2 and had tested the solutions of Equation 2 with the CAS. Following his too early and wordy discourse on Equation 1, T3 had students work briefly on Equation 1, but in fact never asked them how they viewed it at a meta level.

T3 then wrote a transitional equation on the board, $5\left(a-3\right) + 2\left(a-3\right)^2 = 3\left(a-3\right)^3$, and proceeded to illustrate his recommended substitution technique by replacing $\left(a-3\right)$ by x, suggesting that students apply this technique to the solving of Equation 2. This was an adaptation that not only further confounded our initial intentions with respect to students' seeing structural similarities between the two equations, but also presented an added mathematical difficulty for the students: Equation 2 conveying a term in both y and y–2, the substitution of x for y–2 gives either a two-variable equation or a term in x and $x+2$. The transitional equation introduced by T3 did not involve such a hindrance. As the students began working on Equation 2, one did complain that the substitution of x for $y–2$ gave him an xy term, which got him stuck.

A further adaptation by T3 concerned his use of the CAS technology. For the third question, which asked students to check their solutions with those produced by the CAS, T3 chose to eliminate it. Having introduced Equation 2 with a view-screen display of the three solutions yielded by the CAS, he subsequently asked the students to find the same three solutions with paper and pencil. Thus, the surprise realization that there might be three solutions, and how it came to be that one of them had been lost through their paper-and-pencil techniques, was never provoked in T3's class.

DISCUSSION

Our analysis of the spontaneous shaping of the researcher-designed resources indicated adaptive activity not only in all three of the key features of the task-based resources (the mathematics, the students, and the technology) but also in their coordination. Researchers (e.g., Freeman & Porter, 1989) have argued that, if teachers' guides were more explicit and less ambiguous, the degree of closeness between teaching practice with these resources and the intentions of the resource designers could be greater. Our findings are in disagreement with such arguments that suggest that greater detail will necessarily lead to a closer following of curriculum materials. No matter how explicitly expressed the researcher-designers' intentions may be, adaptation of the resources will take place. Our analysis of the nature of the adaptations that were forged with respect to both the implicitly-suggested and explicitly-expressed intentions of the researcher-designers showed that, in both intentional domains, teachers will adapt the resources that they use. Clearly, teachers' past experiences guided the ways they used the resources; however, discussion of this issue is beyond the scope of the present report (for the complete analysis from which this abridged research report is drawn, see Kieran, Tanguay, & Solares, in press).

In closing, our findings regarding the various ways in which teachers adapted the researcher-designed resources cast light on a particular aspect of the theoretical frame of the *documentational approach of didactics*, namely the differential role that the same resources can play within that process of *documentational genesis* whereby resources occasion the shaping of individual teaching practice. The implicit and explicit aspects of the researcher-designed resources served as both affordances and constraints that influenced teachers' activity. Resources are not neutral; they speak to different teachers in different ways – even to teachers using the same resources and sharing the same goal of participating in a research project aimed at developing the technical and theoretical knowledge of algebra students within a CAS-supported environment. It is important to note, however, that the different ways in which the same resources were shaped were by no means irrelevant or insignificant in nature; they either promoted or impeded the emergence of different techniques and theoretical-conceptual elements in students. But that is a whole other story.

Acknowledgments

We express appreciation to A. Boileau, to the participating teachers, and to those who, with C. Kieran and D. Tanguay, collaborated in designing the task-sequences: A. Boileau, F. Hitt, J. Guzmán, and L. Saldanha. We also acknowledge the support of the Social Sciences and Humanities Research Council of Canada (Grant #410-2007-1485), the *Fonds québécois de recherche sur la société et la culture* (FQRSC, Grant #2007-NP-116155), and travel funds from PROMEP/UPN-México (2010).

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