**Name:**

**Activity 5: Sum and Difference of Cubes**

**Part I (with CAS): From factored form to expanded form**

The following factored forms are different from those we’ve already encountered. Use the EXPAND command in your calculator to investigate whether the indicated multiplication of factors produces interesting results.

|  |  |
| --- | --- |
| Factored form | Expanded form displayed by the calculator |
| 1. | *x3 + 8* |
| 2. | *x3 - 27* |
| 3. | *x3 + 343* |
| 4. |  |
| 5. |  |

**Part II (with paper & pencil and CAS): Constructing and testing a general algebraic rule**

II*a)* Notice the form of each expanded result displayed by the calculator. Describe how this form is related to that of the corresponding factors.

The form of the result is a binomial composed of the cube of the first term of the first factor, followed by the sign of the first factor, followed by the cube of the second term of the first factor. It is noted that the second factor has as its form the square of the first term of the first factor, followed by the opposite sign of the first factor and the product of the two terms of the first factor, followed by the same sign as the first factor and the square of the second term of the first factor.

II*b)* State the regularity or patterns that you noticed (across the five examples) in terms of two general algebraic rules.

Rule 1: *(a + b)(a2 – ab + b2) = a3 + b3*

Rule 2: *(a — b)(a2 + ab + b2) = a3 — b3*

II*c)* Use paper and pencil to show that the rules you found in question *b* above work.

*(a + b)(a2 – ab + b2) = a3 — a2b + ab2 + a2b — ab2  + b3 = a3 + b3*

*(a — b)(a2 + ab + b2) = a3 + a2b + ab2 — a2b — ab2 — b3 = a3 — b3*

II*d)* How would you use your calculator to verify the algebraic rules you derived in question *b* above? Use the table below to show your work.

|  |  |
| --- | --- |
| What you enter into the CAS | What the CAS displays for the result |
|  |  |
| EXPAND(*(a + b)(a2 – ab + b2))* | *a3 + b3* |
|  |  |
| EXPAND(*(a — b)(a2 + ab + b2))* | *a3 — b3* |
| OR |  |
| FACTOR(*a3 + b3)*  FACTOR(*a3 — b3)*  OR  *(a + b)(a2 – ab + b2) = a3 + b3*  *(a — b)(a2 + ab + b2) = a3 — b3* | *(a + b)(a2 – ab + b2)*  *(a — b)(a2 + ab + b2)*  True  True |

## Classroom discussion of Parts I and II

##### **Part III (with paper & pencil): From expanded form to factored form**

III(A) Factor each of the following expressions completely, using only paper and pencil. Show all your work in the right-hand column below:

|  |  |
| --- | --- |
| Given expression | Work involved in factoring the given expression |
| 1. | *8u3 - v3 = (2u - v)(4u2 + 2uv + v2)* |
| 2. | *27w3 + 8z3 = (3w + 2z)(9w2 - 6wz + 4z2)* |
| 3. | *(u + 2)3 - (u - 2)3 = ((u + 2) - (u - 2))((u + 2)2 + (u + 2)(u - 2) + (u - 2)2)*  *= (u + 2 - u + 2)(u2 + 4u + 4 + u2 - 4 + u2 - 4u + 4)*  *= 4(3u2 + 4)* |

4. Explain how you used the identities for the sum and difference of cubes to factor the above

expressions.

|  |
| --- |
| I mapped the first term of the given expression on *a3* and the second term of the given expression on  *b3*. For instance, in the second given expression, *27w3* is (*3w*)*3* and *8z3* is *(2z)3*. Then I factored as  if the expression were *a3+b3* or *a3—b3.* I proceeded similarly for the other given expressions.  The third expression above required some additional algebraic manipulation in order to  produce the final factored form. |

III(B) 1. Factor this expression using paper and pencil: .

|  |
| --- |
| *v9 + w9 = (v3)3 + (w3)3*  *= (v3 + w3)(v6 - v3w3 + w6)*  *= (v + w)(v2 - vw + w2)(v6 - v3w3 + w6)* |

2. Which identities helped you to factor the expression in question B 1 above? Please explain how you

applied these identities.

|  |
| --- |
| I first expressed *v9 + w9* as *(v3)3 + (w3)3*. This is now clearly a sum of cubes. So I took the rule  for *a3 + b3* and mapped *v3* onto *a* and *w3* onto *b*. After doing the factorization, I noticed that  the first factor was *v3 + w3*. So I factored that part similarly, mapping the terms onto the same rule. |

3. Factor this expression using paper and pencil: .

|  |
| --- |
| *x6 - y6 = (x3)2 - (y3)2*  *= (x3 + y3)(x3 - y3)*  *= (x + y)(x2 - xy + y2)(x - y)(x2 + xy + y2)*  OR  *x6 - y6 = (x2)3 - (y2)3*  *= (x2 - y2)((x2)2 + x2y2 + (y2)2)*  *= (x - y)(x + y)(x4 + x2y2 + y4)*  *= (x - y)(x + y)(x2 + xy + y2)(x2 - xy + y2)* |

4. Which identities helped you to factor the above expression, ? Please explain how you applied these identities.

|  |
| --- |
| There are two different ways of looking at .  One way is to first look at it as a difference of squares (the first case above), then apply the difference  of squares identity, and then apply both the sum and the difference of cubes rules.  The other way is to first look at it as a difference of cubes (the second case above), then apply the difference of cubes rule, and then apply the difference of squares identity. The fourth degree  polynomial factor can also be factorized completely as shown above. |

## Classroom discussion of Part III, A and B

**Part IV (Homework challenge with paper and pencil): Applying the identities**

Problem 1:

Pierre claims that, “For any two integers whose difference is 2, the difference of their cubes is always an even integer”.

Argue for or against Pierre’s claim. Show your work in the space below.

Let the two integers be represented by *x* and *x - 2*.

The difference of their cubes is *x3 - (x - 2)3*, which, by applying the difference of cubes rule, can be re-expressed as follows:

*x3 - (x - 2)3 = (x - (x - 2))(x2 + x(x - 2) + (x - 2)2)*

*= (x - x + 2)(x2 + x2 - 2x + x2 - 4x + 4)*

*= 2(3x2 - 6x + 4)*

One of the factors of the result is 2. Therefore the result is an even number. Furthermore, as *x* and *x—2* were integers to begin with, the result is an even integer.

Problem 2:

Eric made the following assertion: “Any integer raised to the power of 6, from which 1 is then subtracted, is always divisible by one less than that integer as well as by one more than that integer”.

Argue for or against Eric’s assertion. Show your work in the space below.

Let the integer raised to the power of 6 be represented by *x6*. Then one less than this is *x6 - 1*.

*x6 - 1* can be re-expressed as follows:

*x6 - 1 = (x2)3 - 13*

*= (x2 - 1)(x4 + 1x2 + 1)*

*= (x - 1)(x + 1)(x4 + 1x2 + 1)*

Since two of the factors of *x6 - 1* are *x - 1* and *x + 1*, this shows that *x6 - 1* is always divisible by either of these factors or by both. Furthermore, it is clear that *x - 1* is “one less than the given integer” and that

*x + 1* is “one more than the given integer”.