Activity 2: Continuation of Equivalence of Expressions

**Lesson 3**

Part I (30 minutes): Exploring and interpreting the effects of the ENTER button, and the EXPAND and FACTOR commands

*Notes to teacher*

Some anticipated student difficulties during this activity -- which should be addressed during the discussion following the individual work:

1. In Part I: Relating the structure of given expression 1 with the result of EXPAND.

It may help to relate this to distributivity of multiplication/division over addition

[ ]

2. Managing forms that involve factoring out –1. This will first be encountered in Part I, expression 3. It surfaces again in the homework assignment (Part III) where students might need to do some paper/pencil algebraic manipulation in order to determine the largest common set of admissible values of *x*.

3. Qualifying equivalence. In Part I B, for given expression 4, CAS commands produce simplified forms that “hide” the constraint on admissible values of *x* (i.e., R\{-2}). We therefore expect that students might need guidance in understanding that the equivalence of expressions 1 and 4 is subject to the constraints on *x* for the given expression 4. In Part II A, students will confront this issue again. In this case the common set of admissible values of *x* is R\{-3, 4}. They will confront it yet again in the homework assignment (Part III)

I(A) **(with CAS)** Fill in the table below with the calculator screen display as requested: (Note: the results have been entered in the teacher version)

|  |  |  |  |
| --- | --- | --- | --- |
| **Given expression**  | **Result produced by the ENTER button** | **Result produced by****FACTOR** | **Result produced by EXPAND** |
| 1.  |  |  |  |
| 2. |  |  |  |
| 3.  |  |  |  |
| 4.  |  |  |  |

I(B) **(with paper and pencil)**

1. For given expression 1 (of Part I A): 

* Describe how the structure of each of the 3 forms produced by the calculator compares with that of the given expression.
* Are all three of these forms equivalent to the given expression? Please explain.

2. For given expression 2, , show the algebraic steps you would use to arrive at the form produced by the ENTER button, .

3. Consider the given expression 3, . Use paper-and-pencil algebra to show how to obtain the form produced by the FACTOR command, .

4. Consider the given expression 4, . Use paper and pencil algebra to show how to obtain the form produced by the EXPAND command, .

5. In the table of Part I A, above, which expressions are equivalent to each other (state as many as you can)? Please justify your response. Is this equivalence subject to any constraints on admissible values of *x*? Please explain.

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Classroom discussion of Part I A and B, after individual work: “Which are the equivalent expressions in the above table?“

Two additional issues for teacher to highlight in discussion:

1. All the above CAS commands produce expressions that are equivalent to the given expression, subject to certain constraints. These expressions are thus equivalent to each other.
2. Of the four given expressions, 2 and 3 are equivalent, as are 1 and 4 (subject to the constraint that x ≠ -2). Why? (how did students decide/determine these equivalences?)

**Part II (30 min.)**: **Showing equivalence of expressions by using various CAS approaches**

Here is a list of 4 expressions that are equivalent, subject to certain constraints.

Table 1

|  |
| --- |
| Given expression  |
| 1.  |
| 2.  |
| 3.  |
| 4.  |

II(A) Determine the largest common set of admissible values of *x* for this set of expressions. Show and explain how you determined this (Note: response supplied in the teacher version)

|  |
| --- |
| R \ {-3, 4} |

II(B) Using each of the four methods once and only once, show that all four expressions from Table 1 are equivalent. State what you entered and the CAS results in Table 2.

Note: you need to be strategic in deciding which expression to use with which command.

(You may use the worksheet provided on the last page for keeping track of your work).



Table 2

|  |  |  |
| --- | --- | --- |
| **CAS method** | **What you enter into the CAS** | **Result displayed by the CAS** |
| Test for equality |  |  |
| FACTOR |  |  |
| EXPAND |  |  |
| ENTER |  |  |

II(C) Using only the results in Table 2, prove the six equivalent statements asserted

in Table 3.

Note: You need not fill the cells in the order in which they are presented below.

Table 3 (the symbol “≡” denotes equivalence)

|  |  |
| --- | --- |
| **Asserted equivalence** | **Proof of equivalence** |
| Exp1 ≡ Exp2 |  |
| Exp 1 ≡ Exp3 |  |
| Exp1 ≡ Exp4 |  |
| Exp2 ≡ Exp3 |  |
| Exp2 ≡ Exp4 |  |
| Exp3 ≡ Exp4 |  |

Classroom discussion of Part II A, B, C:

(i) A big idea is to use transitivity of ≡ to show the asserted equivalences in Table 3. Students might, for example, fill in Table 2 as follows (note that we have used our own notation in this display). This approach, or others in a similar vein, would permit them to efficiently justify the six equivalences of Table 3. A key element of this approach is noticing that the expanded form of, say, Expression 1 is the same as the form of the given Expression 3.

Table 2

|  |  |  |
| --- | --- | --- |
| **CAS method** | **What you enter into the CAS** | **Result displayed by the CAS** |
| Test for equality | Exp3=Exp4 | True |
| FACTOR | FACTOR (Exp4) | 3(x-3)(2x-1) |
| EXPAND | EXPAND (Exp1) | Exp3 |
| ENTER | Exp2 (then press ENTER) | 3(x-3)(2x-1) |

(ii) A second issue to raise is the alternative format for a test of equality

(i.e., “Exp3 **–** Exp4 = 0”). Students may want to try out this alternative test at this point.

 **Part III:** **Homework Assignment**

(Aim: to see how students manage to show equivalence when left to their own devices, and whether they notice that the factored and expanded forms for this given set of expressions are identical. Suggested discussion issue: did anyone use FACTOR, as opposed to EXPAND, for all expressions? If so, how do these forms compare?)

**A. Prove that the four expressions in Table 4 are equivalent, by means of whatever CAS approach(es) you wish to use . Show your work in Table 5.**

**Table 4**

|  |
| --- |
| Given expression  |
| 1.  |
| 2.  |
| 3.  |
| 4.  |

**Table 5**

|  |  |
| --- | --- |
| **What you enter into the CAS** | **Result displayed by the CAS** |
|  |  |
|  |  |
|  |  |
|  |  |

Explain how the results in Table 5 above allow you to conclude that the four expressions are equivalent.

|  |
| --- |
|  |

B. Determine the largest common set of admissible values of *x* for this set of expressions. Show how you determined this.

|  |
| --- |
|  |

C. Do you find anything surprising about the factored and expanded forms of this given set of expressions? Please explain.

|  |
| --- |
|  |

**Worksheet for Part II (B)**

|  |  |
| --- | --- |
| What you enter into the CAS | Result displayed by the CAS |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |