**Name:**

**Activity 2: Continuation of Equivalence of Expressions**

**Lesson 3**

Part I: Exploring and interpreting the effects of the ENTER button, and the EXPAND and FACTOR commands

I(A) **(with CAS)** Fill in the table below with the calculator screen display as requested:

|  |  |  |  |
| --- | --- | --- | --- |
| Given expression | Result produced by the  ENTER button | Result produced by  FACTOR | Result produced by EXPAND |
| 1. |  |  |  |
| 2. |  |  |  |
| 3. |  |  |  |
| 4. |  |  |  |

I(B) (with paper and pencil)

1. For given expression 1 (of Part I A): 

* Describe how the structure of each of the 3 forms produced by the calculator compares with that of the given expression.

The form of the expression produced by ENTER is identical.

The form of the expression produced by FACTOR has a product of two linear expressions instead of a sum of simple terms.

The form of the expression produced by EXPAND is a sum of simple terms and therefore differs from the form of the given expression (which consists of a single fraction).

* Are all three of these forms equivalent to the given expression? Please explain.

Yes: all the forms can be re-expressed as the given expression by using algebraic manipulations.

For example, the expanded form is the result of applying the distributive property to the given expression:  is equivalent to  which in turn yields.

We can also prove the equivalence of these forms by using a CAS test of equality (as in the Part IV of Activity 1).

2. For given expression 2,  , show the algebraic steps you would use to arrive at the form produced by the ENTER button, .

[*(x-2)2+(7x-2)(x-2)*]*/4 =* [*(x-2)(x-2)+(7x-2)(x-2)*]*/4*

*= (x-2)*[*(x-2) + (7x-2)*]*/4*

*= (x-2)(8x-4)/4*

*= 4(x-2)(2x-1)/4*

*= (x-2)(2x-1)*

3. Consider the given expression 3, . Show with paper-and-pencil algebra how to obtain the form produced by the FACTOR command, .

*(2-x)(1-2x) = -1(x-2)(2x-1)(-1)* factor (-1) out of each of the given expressions

*= (x-2)(2x-1)* the product of (-1)(-1) yields 1

4. Consider the given expression 4, . Show with paper-and-pencil algebra how to obtain the form produced by the EXPAND command, .

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= 

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= 

5. In the table of Part I A above, which expressions are equivalent to each other (state as many as you can)? Please justify your response. Is this equivalence subject to any constraints on admissible values of *x*? Please explain.

Expression 1 and expression 4 are equivalent, since the results of FACTOR (and EXPAND) show that they are expressible in a common form. This equivalence is restricted to all real numbers other than x = -2 (since expression 4 is undefined there).

Similarly, expression 2 and expression 3 are equivalent (since ENTER and FACTOR both yield identical forms, and similarly for EXPAND). This equivalence is not subject to any restrictions.

In addition, as discussed in IB1, all forms of a given expression produced by Enter, Factor, and Expand are equivalent to each other.

## Classroom discussion of Part I A and B

**Part II**: **Showing equivalence of expressions by using various CAS approaches**

Here is a list of four expressions that are equivalent, subject to certain constraints.

Table 1

|  |
| --- |
| Given expression |
| 1. |
| 2. |
| 3. |
| 4. |

II(A) Determine the largest common set of admissible values of *x* for this set of expressions. Show and explain how you determined this.

|  |
| --- |
| By definition, the set of admissible values for a given expression in *x* consists of those values of *x* for which the expression defines a real number.  The largest common set of admissible values of *x* for these 4 expressions consists of all real numbers on which all of them are defined: R-{-3, 4}. This set is determined by considering values of *x* on which each expression does not define a real number. In the case of these four expressions, this occurs whenever a denominator equals zero (since division by zero is undefined): that is when 7x+21 = 0 and when x+3 = 0, which implies x = -3, and when x - 4= 0, which implies x = 4. |

II(B) Using each of the four methods once and only once, show that all four expressions from Table 1 are equivalent. In Table 2, state what you entered and the CAS results.

Note: you need to be strategic in deciding which expression to use with which command.

(You may use the worksheet provided on the last page for keeping track of your work)

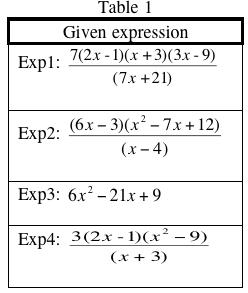


Table 2

|  |  |  |
| --- | --- | --- |
| **CAS method** | **What you enter into the CAS** | **Result displayed by the CAS** |
| Test for equality | Exp3=Exp4 | True |
| FACTOR | FACTOR (Exp4) | *3(x-3)(2x-1)* |
| EXPAND | EXPAND (Exp1) | Exp3 |
| ENTER | Exp2 (then press ENTER) | *3(x-3)(2x-1)* |

II(C) Using only the results in Table 2, prove the six equivalent statements shown in Table 3 (subject to the constraints established in IIA).

Note: You need not fill the cells in the order in which they are presented below.

Table 3 (the symbol “≡” denotes equivalence)

|  |  |
| --- | --- |
| **Asserted equivalence** | **Proof of equivalence** |
| Exp1 ≡ Exp2 | i) Exp1 ≡ Exp3 (since the l.h.s. expands to the same form as the r.h.s.), and  ii) Exp3 ≡ Exp4 (by the test of equality), and  iii) Exp4 ≡ Exp2 (since they are re-expressible in a common form)  Thus, we have Exp1 ≡ Exp2 (by transitivity). |
| Exp 1 ≡ Exp3 | The expanded form of expression 1 is the same as the given form of expression 3. |
| Exp1 ≡ Exp4 | i) Exp1 ≡ Exp3 (since the l.h.s. expands to the same form as the r.h.s.), and  ii) Exp3 ≡ Exp4 (by the test of equality).  Thus, we have Exp1 ≡ Exp4 (by transitivity). |
| Exp2 ≡ Exp3 | Exp2 ≡ Exp4 (via common form), and Exp4 ≡ Exp 3 (by equality test).  Thus, Exp2 ≡ Exp3 (by transitivity). |
| Exp2 ≡ Exp4 | Both expressions have the common form *3(x-3)(2x-1)*. |
| Exp3 ≡ Exp4 | By the equality test, which displays “true”. |

**Classroom discussion of Part II A, B, and C**

**Homework Assignment**

**A.** Prove that the four expressions in Table 4 are equivalent, by means of whatever CAS approach(es) you wish to use. Show your work in Table 5.

**Table 4**

|  |
| --- |
| Given expression |
| 1. |
| 2. |
| 3. |
| 4. |

**Table 5**

|  |  |
| --- | --- |
| **What you enter into the CAS** | **Result displayed by the CAS** |
| EXPAND () |  |
| EXPAND () |  |
| EXPAND () |  |
| EXPAND () |  |

Explain how the results in Table 5 above allow you to conclude that the four expressions are equivalent.

|  |
| --- |
| All given expressions are equivalent because they all expand to the same common form. |

**B.** Determine the largest common set of admissible values of *x* for this set of expressions. Show how you determined this.

|  |
| --- |
| The largest common set of admissible values of *x* for this set of expressions is all real  numbers other than those where any expression is undefined (that is, where any denominator  equals zero):  R-{1/3, 5/4, 3}  Exp1 is undefined at the values *x* = 3 and *x* = 5/4, by inspection of its denominator.  Exp2 is undefined at the value *x* = 3, also by inspection of its denominator.  Exp3 is undefined at the values *x* = 3 and *x* = 1/3, since its denominator factors to *-1(3x2-10x+3)*  and then to *–1(3x-1)(x-3)*.  Exp4 is undefined at the values *x* = 3 and *x* = 1/3, since its denominator factors to  *4(x2-2x+1)- (x2+2x+1) = 4x2-8x+4-x2-2x-1*  *= 3x2-10x+3*  *= (3x-1)(x-3)* |

**C.** Do you find anything surprising about the factored and expanded forms of this given set of expressions? Please explain.

|  |
| --- |
| From Table 5, we note that the expanded form is always .  If we use the FACTOR command with each expression, we note that the factored form of each  is always .  It seems unusual that the factored and expanded forms are identical, but this is sometimes the  case with certain types of expressions. |

**Worksheet for Part II (B)**

|  |  |
| --- | --- |
| What you enter into the CAS | Result displayed by the CAS |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |