Activity 3: Transition from Expressions to Equations

Point of insertion: Within 1 week of concluding Activity 2, or immediately after Activity 2.

**Note to teacher:** This transitional activity is fundamental. The crux of the intended transition is to embed equations/equation-solving within the idea of equivalence/non-equivalence of algebraic expressions, so that the former is characterized in terms of the latter. Such a perspective is often neglected in a traditional, skills-based approach to equation solving. Note that the teacher needs to bring to center stage the close connection between equivalent/non-equivalent expressions and equations/equation-solving, and be sensitive to the conceptual difficulties entailed in making such a connection.

**Part I (with CAS): Introduction to the use of the SOLVE command** (10 minutes)

Recall in our first activity on equivalence of expressions that we encountered expressions that were not equivalent (a reminder of the definition of equivalence: “If, for any admissible number that replaces *x*, each of the expressions gives the same value, we say that these expressions are equivalent on the set of admissible values.”).

With those non-equivalent expressions, when we entered into the CAS the equations formed with such expressions, the CAS did not display “true”. This was because there are only *some* (or *no*) values of x that, when substituted into both sides of the equation, produce equal results. In the present activity we will use the CAS to find the values of x that produce equal results for given expressions.

Here is an example of two expressions that are clearly not equivalent: *x*2 and *x.*

If we enter into the calculator an equation formed of these two expressions (*x*2= *x*), it will therefore not display “true”. To find those values of x for which the two expressions produce equal values, we can use the SOLVE command of the CAS.

**Syntax**: SOLVE (Expr1 = Expr2, x), presuming that “x” is the variable name that appears in each expression, and Expr1 and Expr2 represent the given expressions.

**Solve the equation *x*2= *x* using the SOLVE command in your CAS.**

1. What does the CAS display as a result?
2. Anticipate what the calculator would display were you to substitute each of these x-values back into the equation?
3. Using your CAS’s “with operator” (“ **|**”), verify that the calculator indeed

displays what you expected in Question 2.

**Syntax**: Expr1=Expr2 **|** x=*value*

**Terminology**: The values of x for which both expressions produce equal results are commonly referred to as “solutions” to the equation.

Part II (with CAS, 20 minutes):

Expressions revisited, and their subsequent integration into equations

Here are three expressions:

1. x(x2-9)
2. (x+3)(x2-3x)-3x-3
3. (x2-3x)(x+3)

II(A) Use your calculator to determine which of these expressions are equivalent. Fill in the table below with the appropriate information.

|  |  |  |
| --- | --- | --- |
| What I entered into the CAS | What the CAS displays | My interpretation of what the CAS displays |
|  |  |  |
|  |  |  |
|  |  |  |

II(B) Which expressions are equivalent? Which are not equivalent? Please explain.

II(C) Construct an equation using one pair of the given expressions that are not equivalent (see Part II B, above). Use your calculator to determine those values of x, if any, for which both expressions in your equation are equal.

|  |  |
| --- | --- |
| What I entered into the CAS | What the CAS displays |
|  |  |
|  |  |

II(D) How would you use the CAS to verify that the values you found for x (in Part II C, above) are solutions to your equation? Fill in the table below with the appropriate information.

|  |  |
| --- | --- |
| What I would enter into the CAS | The result that the CAS would display |
|  |  |

II(E) Construct an equation from another pair of the given expressions that are **not equivalent** (see Part II B. above). Without using the CAS and without using paper and pencil algebra (use only a logical argument), determine the solution(s) to this equation. Please explain.

II(F) Construct an equation using a pair of the given expressions that **are equivalent** (see Part II B, above). Without using your CAS and without using paper and pencil algebra (use only a logical argument), determine the solution(s) of this equation. Please explain.

# Classroom discussion of Parts I and II

**Whole-class discussion**: Expr1 and Expr3 are equivalent. For Question E, it will be of interest to see whether students realize that they need not solve more than one equation involving pairs of non-equivalent expressions (i.e., either SOLVE Expr1=Expr2 or SOLVE Expr2 = Expr3). Similarly, (in Question F) students should not have to solve Expr1=Expr3 in order to realize that all values of x satisfy this equation.

## Part III (paper & pencil, 15 minutes): Constructing equations and identities

**III(A)** 1. Construct an equation made from two equivalent expressions of your own choosing.

2. Explain your reasons for choosing these two particular expressions.

3. Without solving it, what can you say about the solutions of this equation?

4. How would you use the CAS to support your response to Question A3 just above?

**III(B)** 1. Construct an equation made from two non-equivalent expressions of your own choosing.

2. Explain your reasons for choosing these two particular expressions.

3. What can you say about the solutions of this equation?

4. How would you use the CAS to support your response to Question B3 just above?

# Classroom discussion of Part III, A and B

**Whole-class discussion:** Have various students share their responses and thinking on Questions A and B, # 1-4. In particular, it will be of interest to know how students interpret what they think the CAS would display in Questions 4. (e.g., What does “true” signify for the solution to an equation formed from two equivalent expressions in Question A4? Similarly, for the case in Question B4).

Part B raises special issues: in questions B3 and B4, there are two possibilities for anticipated solutions and for supporting with CAS, respectively. That is, for equations formed from non-equivalent expressions, two cases are possible: equations true for *some* values of *x* versus equations having no solution. This might be an appropriate juncture at which to employ graphical representations to sharpen distinctions between these two cases. However, since this issue arises as well in Part IV, it may be more opportune to refer to graphical representations at that later time.

Introduce the following terminology: an equation formed from two equivalent expressionsis commonly referred to as an **identity** (that is, left and right sides are identically equal).

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**Part IV (with CAS, 15 minutes): Synthesis of various equation types**

1. Solve the following equations using the SOLVE command of the CAS (3 types of equations: 2 conditional statements, 1 identity, and 1 contradiction).

|  |  |
| --- | --- |
| Given equation | What the CAS displays |
| 1. (2–x)2 = x(2x–4) |  |
| 2. (x–5)(3x+7)–5 = 3x2-8x–40 |  |
| 3. 3x2–x–1 = 2x+5 |  |
| 4. |  |

2. How do you interpret each of the CAS displays in Question 1 above?

3. What does the nature of an equation’s solution(s) indicate about the equivalence or non-equivalence of the expressions that form the equation?

### Classroom discussion of Part IV

**Whole-class discussion**: Part IV synthesizes the three different types of equations to which students have been introduced: those that are true for some, true for all, and true for no replacement values for x (contradictions). Here, students interpret CAS displays relative to solving each type—contradictions, in particular. Students may need to see other, perhaps simpler (more evident by inspection), examples of contradictions. It will be useful, as a concluding point, to have students interpret equations that are contradictions (i.e., true for none) in terms of equivalence; that is, left and right sides of the equation are non-equivalent expressions that will never yield equal values, no matter what replacement for x is used. This should be contrasted with the other two cases that they will have already encountered prior to this point. (i.e., left and right sides yield equal values for all replacements of x versus left and right sides yield equal values for only some replacements of x).

Possible homework assignment: additional examples of the types given in Part IV, if teacher feels that students will benefit.