Name:

Date:

Activity 8: Systems of Equations

**Lesson 1**: **An introduction to systems of equations**

**Part I (with CAS): Using numerical evaluation to test solutions of given types of equations**

(A) First-degree equations in a single unknown

1. The following table contains an equation and some numerical values.

*Without solving* this equation, determine whether the values of the left-hand column are solutions of the equation (using the CAS). But before doing so, describe and justify the strategy you will use to determine whether a given number is a solution (write in the box below). What information will your CAS strategy provide you with?

2. Now the work with the calculator (but no solving, please). Fill in the table below with the appropriate information.



|  |  |  |
| --- | --- | --- |
| Values for *x* | What I entered into the CAS | Result displayed by the CAS |
| *x* = -2 |  |  |
| *x* = 2 |  |  |
| *x* = -5 |  |  |

3. Are there other solutions for this equation? If so, please give one and justify your choice

(B) First-degree equations in two unknowns

1. The next table contains a new equation and some numerical values.

*Without solving* this equation, but again using the CAS, determine whether each pair of values in the left-hand column is a solution. But before doing so, describe and justify the strategy you will use to determine whether a given number is a solution (write in the box below). What information will your CAS strategy provide you with?

2. Now the work with the calculator (but no solving, please). Fill in the table below with the appropriate information.



|  |  |  |
| --- | --- | --- |
| Values for the pair *x* and *y* | What I entered into the CAS | Result displayed by the CAS |
| *x* = 3 and y = 12 |  |  |
| *x* = -3 and y = 4 |  |  |
| *x* = -18 and y = -6 |  |  |

3. Are there other solution pairs for this equation? If so, please give one and justify your choice

# (C) Systems of two first-degree equations in two unknowns

1. The next table contains a system of equations and a few ordered pairs of numbers.

*Without solving* this system of equations, determine whether each pair in the left-hand column is a solution of the system (using the CAS). But before doing so, describe and justify the strategy you will use to determine whether a given number is a solution (write in the box below). What information will your CAS strategy provide you with?

2. Now the work with the calculator (but no solving, SVP). Please write in the table below what you entered into the calculator, as well as what the calculator displays as a result.



|  |  |  |
| --- | --- | --- |
| Values for the pair *x* and *y* | What I entered into CAS | Result displayed by CAS |
| *x* = 0 and *y* = 2 |  |  |
| *x* = 4 and *y* = 3 |  |  |
| *x* = 2 and *y* = 1 |  |  |

3. Are there other solutions for this system of equations? If so, please give one and justify your choice.

|  |
| --- |
|  |

4. Are there any questions or ideas that occurred to you while you were working on these three types of equations? What were they?

###### Classroom discussion of Part I

[For note-taking during the class discussion]

Section A equation: 

Section B equation: 

Section C system of equations: 

**Part II (with CAS):**

**Interpreting CAS solutions for equations in one and two unknowns**

II (A) Solving an equation in one unknown.

Use the command “SOLVE” of the CAS to solve the following equation:

*4(3x-7) = 2(3-x)+5*

|  |  |
| --- | --- |
| What you enter into the CAS | What the CAS displays for the result |
|  |  |

II (B) Solving an equation in two unknowns.

The six following questions concern the equation: *2x+7 = 8y+11*.

1. What do you predict as a result if you use the CAS to solve this equation for *x* ?

|  |
| --- |
|  |

2. Use the CAS to solve this equation for *x*:

|  |  |
| --- | --- |
| What you enter into the CAS | What the CAS displays for the result |
|  |  |

3. How do you interpret the result displayed by the CAS?

|  |
| --- |
|  |

4. What do you predict for the result if you use the CAS to solve this equation for *y*?

|  |
| --- |
|  |

5. Use the CAS to solve the equation *2x+7 = 8y+11* for *y:*

|  |  |
| --- | --- |
| What you enter into the CAS | What the CAS displays for the result |
|  |  |

6. How do you interpret the result displayed by the CAS?

|  |
| --- |
|  |

II (C) Distinctions between solutions of equations in one and two unknowns

1. You probably noticed that, in Part II (A), the CAS displayed a numerical value as the solution for x. In contrast, for Part II (B), the calculator displayed the result for *x* in the form of an algebraic expression. How do you explain this difference?

|  |
| --- |
|  |

2. How can we use these expressions displayed by the CAS to find numerical solutions of the equation *2x + 7 = 8y + 11*?

|  |
| --- |
|  |

##### **Classroom discussion of Part II A, B, C**

II (D) Using CAS to generate and verify solutions to equations in two unknowns

1. Use the CAS to generate three solutions for each of these equations (keep track of what you enter into the CAS as you go along, and of what the CAS displays as a result at each step). Verify at least one solution for each equation (with the CAS).

(a) 

|  |  |
| --- | --- |
| What I entered into the CAS | Result displayed by the CAS |
|  |  |

(b) 

|  |  |
| --- | --- |
| What I entered into the CAS | Result displayed by the CAS |
|  |  |

2. Give at least one question or idea that occurred to you while doing Part II D above (for example, a question concerning difficulties you experienced).

**Lesson 2** (Parts IIIA, IIIB, IIIC)

**Part IIIA (paper & pencil): Review of Comparison and Substitution methods**

1. Here is the COMPARISON method for solving a system of linear equations (adapted from p. 126 of your textbook):

|  |  |
| --- | --- |
| COMPARISON METHOD  The algebraic comparison method consists of: | x + 3y = 5  7x + 6y = 20 |
| 1. Isolating the same unknown in each of the equations, thereby creating two expressions each in a common single unknown; | y = (5-x)/3  y = (20-7x)/6 |
| 2. Setting the two expressions obtained in step 1 equal to each other to construct an equation in one unknown; | (5-x)/3 = (20-7x)/6 |
| 3. Solving the resulting equation; | (5-x)/3 = (20-7x)/6  2(5-x) = (20-7x)  10-2x = 20-7x  7x-2x = 20-10  5x = 10  x = 2 |
| 4. Replacing the resulting value in one of the system’s equations to calculate the value of the other unknown of the solution pair. | y = (5 – x)/3 = (5 – 2)/3 = 1  Solution pair is therefore (x, y) = (2, 1)  Verify this! |

Question: Why do you think that this method is called the Comparison method (in other words, in what sense is a comparison being made in this method)?

2. Here is the SUBSTITUTION method for solving a system of linear equations (adapted from p. 128 of your textbook)

|  |  |
| --- | --- |
| SUBSTITUTION METHOD  The algebraic substitution method consists of: | 2x + 3y = 25  5x + y = 30 |
| 1. Isolating, if necessary, one of the unknowns in one of the equations; | y = 30 – 5x |
| 2. Substituting the expression obtained in step 1 for the appropriate unknown in the other equation, thereby creating an equation in a single unknown; | 2x + 3(30 – 5x) = 25 |
| 3. Solving the equation obtained in step 2; | 2x + 90 – 15x = 25  -13x = 25-90  -13x=-65  x = 65/13 |
| 4. Substituting the value obtained in one of the system’s equations to calculate the value of the other unknown in the ordered pair solution. | y = 30 – 5(65/13)  = 65/13  Solution pair is (x, y) = (65/13, 65/13)  Verify this! |

Question: Why do you think that this method is called the Substitution method?

|  |
| --- |
|  |

3. In what way do these two methods (the Comparison and the Substitution methods) allow us to reduce the given situation to one that we already know how to handle?

|  |
| --- |
|  |

# **Classroom discussion of Part IIIA**

### Part IIIB (with CAS): Using the Comparison method with the CAS

Here is a system of linear equations: 

1. With the CAS, use the Comparison method to solve this system (keep track of what you enter into the calculator as you go along, and of what the calculator displays in response to your commands).

|  |  |  |
| --- | --- | --- |
| The Comparison method involves: | What you enter into the CAS | What the CAS displays for the result |
| 1. Isolating the same unknown in each of the equations, thereby creating two expressions each in a common single unknown |  |  |
| 2 & 3. Setting the two expressions obtained in step 1 equal to each other to construct an equation in one unknown; solving the resulting equation |  |  |
| 4. Replacing the resulting value in one of the system’s equations to calculate the value of the other unknown of the solution pair |  |  |

2. How do you verify with the CAS that your solution is correct?

|  |
| --- |
|  |

3. In Step 4 of Question 1 above, you replaced the value obtained in Step 3 (for the first unknown) in one of the equations. Now substitute this same value obtained in Step 3 into the other equation. What do you notice? Why is this so?

|  |
| --- |
|  |

# **Classroom discussion of Part IIIB**

### Part IIIC (with CAS): Using the Substitution method with the CAS

1. With the CAS, use the Substitution method to solve this system (keep track of what you enter into the calculator as you go along, and of what the calculator displays in response to your commands).



|  |  |  |
| --- | --- | --- |
| The Substitution method involves: | What you enter into the CAS | What the CAS displays for the result |
| 1. Isolating, if necessary, one of the unknowns in one of the equations; |  |  |
| 2. Substituting the expression obtained in step 1 for the appropriate unknown in the other equation, thereby creating an equation in a single unknown |  |  |
| 3. Solving the equation obtained in step 2 |  |  |
| 4. Substituting the value obtained in one of the system’s equations to calculate the value of the other unknown in the ordered pair solution. |  |  |

2. How do you verify with the CAS that your solution is correct?

|  |
| --- |
|  |

3. Which of the two methods (COMPARISON and SUBSTITUTION) do you prefer and

why?

|  |
| --- |
|  |

4. What do these two methods have in common (don’t just rewrite the steps of the two methods)?

|  |
| --- |
|  |

# **Classroom discussion of Part IIIC**

### Homework

With the CAS, use the more appropriate method (comparison or substitution) to solve the following systems of linear equations:

(1) *y + 1 = x + 6*

*y – 4 = -x + 3*

(2) *3x + y = 23*

#### 2x + 3y = 48