**Activity 5: Sum and Difference of Cubes**

(Enrichment activity on factoring)



This activity is intended to be introduced in the MATH 436 course, just after page 184 of Chapter 3 of the Reflections 436 textbook

The activity’s aim is to support students’ abilities to deal with algebraic expressions involving the sum and difference of powers that are multiples of 3.

Preamble : 10 minute whole-class discussion, before starting Part I of the activity

* Teacher discusses the items summarized in the table on page 184. After discussing the items in the last row of the table (difference of squares), the teacher will stress moving in the reverse direction. That is, starting from factored forms, have students produce a difference of squares (a few examples where students multiply out factors using paper and pencil to arrive at the difference of squares, with the aim that they see that certain terms cancel out).
* Teacher will ask students to try and generate factors whose product is a sum or sums of squares. The aim is to have students conclude that they can produce forms like x2-y2,but not x2+y2.

## Part I (with CAS, 10 minutes): From factored form to expanded form

The following factored forms are different from those we’ve already encountered. Use the EXPAND command in your calculator to investigate whether the indicated multiplication of factors produces interesting results.

|  |  |
| --- | --- |
|  Factored form |  Expanded form displayed by the calculator  |
| 1.  |  |
| 2.  |  |
| 3.  |  |
| 4.  |  |
| 5.  |  |

**Part II (paper & pencil/CAS, 15 minutes): Constructing and testing a general algebraic rule**

II*a)* Notice the form of each expanded result displayed by the calculator. Describe how this form is related to that of the corresponding factors.

II*b)* State the regularity or patterns that you noticed (across the five examples) in terms of two general algebraic rules.

II*c)* Use paper and pencil to show that the rules you found in question *b* above work.

[Anticipated rules:  and ]

II*d)* How would you use your calculator to verify the algebraic rules you derived in question *b* above? Use the table below to show your work.

|  |  |
| --- | --- |
| What you enter into the CAS  | What the CAS displays for the result |
| Some anticipated student strategies : |  |
| 1) EXPAND() |  |
| 2) =(a3+b3) |  |
| 3) —( a3+b3)= 0 |  |
| 4) FACTOR(a3+b3) |  |

## Classroom discussion of Parts I and II

## Whole-class discussion (to be conducted after completion of Parts I and II): (10 minutes)

* What interesting or remarkable results, if any, did you notice in Part I?
* Discussion of Part II *a)*: What did you notice with respect to the regularity of the factors in this group of expressions? What regularity do you see between these factors and their corresponding expanded forms produced by the calculator?
* Discussion of Part II *b)*: What general algebraic rules did you generate to describe the regularity that you noticed in Part II *a)*?

[,]

Such rules are often referred to as **identities** (i.e., equations formed from equivalent expressions).

[We anticipate that one possible challenge for students will be to represent this regularity using an algebraic rule]

* Discussion of Part II *c)*: How can you show that the rules you generated in Part II *b)* work?

[issues to highlight: (i) why do terms in the intermediate expressions cancel out?, (ii) how is this

reason related to what happens in the case of the difference of squares?]

* Discussion of Part II *d)*: What strategies did you use to answer Question II *d)*? Justify your choice of strategies. What is your interpretation of what the calculator displayed?

##### **Part III (with paper & pencil and CAS, 20 minutes, to be completed for homework if necessary)**

**III(A)** Factor each of the following expressions completely, using only paper and pencil. Show all your work in the right-hand column below:

|  |  |
| --- | --- |
| Given expression | Work involved in factoring the given expression |
| 1.  |  |
| 2.  |  |
| 3.  |  |

4. Explain how you used the identities for the sum and difference of cubes to factor the above

expressions.

|  |
| --- |
|  |

**III(B)** 1. Factor this expression using paper and pencil: .

|  |
| --- |
|  |

2. Which identities helped you to factor the expression in question B 1 above? Please explain how you

applied these identities.

|  |
| --- |
|  |

3. Factor this expression using paper and pencil: .

|  |
| --- |
|  |

4. Which identities helped you to factor the above expression, ? Please explain how you applied these identities.

|  |
| --- |
|  |

## Classroom discussion of Part III, A and B (about 10 minutes)

Discussion of the decomposition and factorization of expressions involving binomials that contain exponents that are even and/or multiples of 3.

Our particular interest with respect to questions A(4), B(2), and B(4) is in knowing how students map the given expression onto the two identities of this lesson. The classroom discussion should draw out students’ ways of thinking about these mappings.

Specific discussion issues:

* In discussion of Part A, have a couple of students show their work and explain the structure they see in the given expressions relative to the identities. For example, what identifications did they make between expression 1, say, and the standard form *a*3-*b*3?
* In discussion of Part B, seeing the structure of the identity in these expressions is complexified due to the nature of the exponents. Allow students to share what structure they see in expressions in B(1) and B(3). In particular, do they see the expression in B(3) as both a difference of squares and a difference of cubes?

• “What kinds of binomials involving exponents are we now able to factor?” Possibly provide

a few examples, like those in B(1) and B(3) of Part III.

**Part IV (Homework challenge with paper and pencil): Applying the identities**

The aim of this part is to deepen the identities’ relevance for students by providing them with opportunities to extend and apply the identities in a numerical and algebraic context. This part also entails an algebraic modeling component.

Problem 1:

Pierre claims that, “For any two integers whose difference is 2, the difference of their cubes is always an even integer”.

Argue for or against Pierre’s claim. Show your work in the space below.

Problem 2:

Eric made the following assertion: “Any integer raised to the power of 6, from which 1 is then subtracted, is always divisible by one less than that integer as well as by one more than that integer”.

Argue for or against Eric’s assertion. Show your work in the space below.

Whole-class discussion (to be conducted the next day after completion of the

Homework Challenge of Part IV) : (20 minutes)

Problem 1: “With regard to the first problem, was Pierre right? How did you show whether Pierre was right or wrong?”

* If strategies that students present are largely numerical, ask the class whether anyone employed an algebraic strategy. Invite those students to share their approaches with the rest of the class.
* Overarching anticipated difficulties (and issues to highlight during discussion):
1. Expressing one quantity in terms of another (“two integers whose difference is 2”), with the aim of producing a cubic algebraic expression containing a single variable;
2. Interpreting the final result of the expansion of the cubic expression as an even number;
3. Seeing algebra as a useful tool for proving something mathematical.

• “What do the numerical strategies buy us? What do the algebraic strategies buy us?”

(**Note**: These questions are intended to query the following: Are students more convinced by numerical examples or by algebraic/general representations of these numerical relationships?)

* “Did anyone use the calculator in this problem? How was the calculator used?”

Problem 2: “With regard to the second problem, was Eric right? How did you show whether Eric was right or wrong?”

* If strategies that students present are largely numerical, ask the class whether anyone employed an algebraic strategy. Invite those students to share their approaches with the rest of the class.
* Overarching anticipated difficulties (and issues to highlight during discussion):
1. After the statement “An integer raised to the power of 6, from which 1 is then subtracted, is always divisible by one less than that integer as well as by one more than that integer” has been translated into algebraic form, what to do with this form?
2. A lack of realization that factors are also divisors might lead to/necessitate an extended discussion of this fact and a return to arithmetic contexts.