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CONNECTING THEORY AND PRACTICE: RESULTS FROM THE TEACHING OF LOGO

ABSTRACT. This paper is concerned with pupils' learning of the concept of variable in the Logo programming environment and how this relates to teaching. Results from three research projects are described and compared in a discussion of how improvements in learning have been influenced by a refinement in teaching method. This refinement has been influenced, in part, by the theories of Vygotsky. Within this paper, details of the changes in teaching approach are described together with a discussion of the effects of these changes. The main conclusion drawn is that in mathematics education we need to make more explicit the underlying theories influencing our work, because these theories influence both the ways in which we work in the classroom and the ways in which we analyse our data.

This makes me think that there is no true teaching other than the teaching which succeeds in provoking in those who listen an insistence — this desire to know which can only emerge when they themselves have taken the measure of ignorance as such — of ignorance inasmuch as it is, as such, fertile — in the one who teaches as well. (Lacan, 1978, p. 242)

INTRODUCTION

Mathematics education research in the UK tends to be rather a-theoretical in nature. This, I believe, is because those of us working in the area find it difficult to whole-heartedly embrace one theoretical discipline, ending up with a fragmented collection of theories with which we attempt to inform our classroom research. Psychological theories seem inadequate because they do not account for the dynamic learning situation within the classroom; sociological theories seem inadequate because they do not account for individual learning.

Rogoff has pointed out that "development is made up of both individual efforts or tendencies and the larger sociocultural context in which the individual is embedded and has been since before conception. Thus biology and culture are not viewed as alternative influences but as aspects of a system in which individuals develop" (Rogoff, 1991, p. 70). However, there is no simple step from accepting this position to finding an appropriate theory to inform classroom research. And if, and when, we find such a theoretical basis it is even more difficult to link theory to the practice of research. The complexity of the situation seems to result in a divided research community — those who expound theory and stay away from classroom practice, and those who carry out research in the

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classroom with very little theoretical foundation. This is unfortunate, because without an adequate theoretical basis we cannot move forward together. Our research efforts will be as fragmented as the learning of pupils in the classroom.

Another aspect of this issue is that many of us in the UK mathematics education research community have ourselves been teachers and inevitably have our own strong implicit and practice-based theories about teaching and learning. These implicit theories need to be critically examined and integrated into a more explicit theoretical framework with a consideration of how much they are influenced by established orthodoxies in the mathematics education world. The established orthodoxies tend to dictate what constitutes "good" teaching. As Desforges and Cockburn have pointed out, "Mathematics education researchers often criticize teachers for asking their pupils 'testing' questions for which they already know the answers and then paradoxically behave themselves as if they know the answers about which questions should be asked" (Desforges and Cockburn, 1987). One aspect of education research is to disentangle the relationship between teaching and learning. In my opinion, we cannot be so certain about what constitutes "good teaching". Perhaps the certainty within the community reflects the insecurity of those who, once removed from classroom practice, are forced into positions of becoming false experts.

In this paper I shall discuss three research projects which have predominantly been concerned with the learning of mathematics in computer environments. One "by-product" of this work has been an investigation of how pupils learn to use variables to write general procedures in Logo (Sutherland, 1987, 1988). My observation is that the pupils (aged 12) who are part of the ongoing Project AnA (Sutherland, 1990) are more competent at writing general procedures in Logo than were the pupils at age 14 of the first project, the Logo Maths Project (Hoyles and Sutherland, 1989). My conjecture is that this is due to a refinement in teaching methods. This paper sets out to describe the nature of these improvements in learning, analysing how they might have been influenced by changes in teaching.

THE LOGO MATHS PROJECT

The first of these projects, the Logo Maths Project, was a three-year longitudinal study investigating the way Logo can be used as an aid to pupils' thinking and learning of mathematics. The theoretical rationale for the study was that pupils learn mathematics through active construction of their own knowledge and that this can be facilitated in a computer environment through an iterative process of conjecture and feedback. In this respect, we were influenced by the constructivist theories of Piaget. At the beginning of the project, we emphasized the importance of pupils' constructing their own knowledge and de-emphasized the role of the teacher in structuring the classroom situation. Pupils initially learned Logo in a relatively open-ended situation, choosing their own goals and their own means of solving these goals.

The methodology used can be described as a mixture of ethnomethodology and case study (Goetz and LeCompte, 1984). Three of us, as researchers, acted as participant observers, sometimes teaching and sometimes making research notes. We carried out case studies of four pairs of pupils over a period of three years, making video recordings of all the Logo sessions for these case-study pupils. All of these recordings were transcribed and so the teacher interventions became available for analysis.

What became increasingly clear, as the project progressed, was that although we had thought that we were keeping our interventions to a minimum, this was not how it appeared when we analysed the transcripts. It is not possible to describe all the details of the research here, but I shall summarize what I consider to be the most important findings from the point of view of teaching and learning.

During the Logo Maths Project, we were influenced by previous research studies (for example, Booth, 1984, and Küchemann, 1981) which suggested that pupils have considerable difficulty with the use and understanding of variables in algebra; so we assumed that pupils would have similar difficulties with the use of variables in Logo. We waited patiently throughout the first year of the project for pupils to choose goals within which we could introduce the idea of variable (consistent with our approach to learning). This hardly ever happened. Pupils predominantly chose projects which focused on the generation of a specific shape (see, for example, Figure 1), and introducing them to the idea of generalizing from the specific did not fit well with their own goals. The transcripts taken from the first year of the project contain many inappropriate teacher interventions to encourage this use of

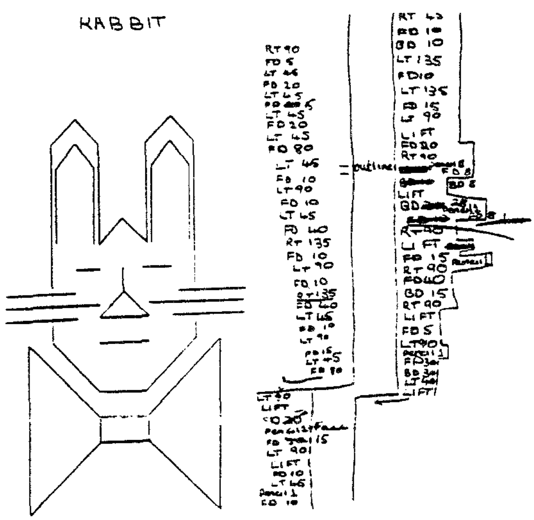


Figure 1. The rabbit.

variables within the pupils' own projects.

Towards the end of the first year of the Logo Maths Project, we changed our strategy and intervened with teacher-directed tasks which introduced pupils to the idea of variable in Logo. This change in direction resulted from our ongoing analysis of the data — that is, the finding that pupils were not choosing projects in which it was appropriate to introduce the idea of variable. We were now structuring the situation for the pupils. This is illustrated by the following excerpt in which two pupils are working on the task of generating a spiral image in Logo.

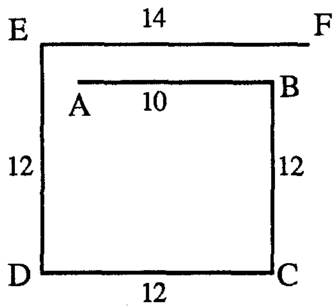


Figure 2. Sally and Janet's spiral.

The teacher interventions are aimed at introducing Sally and Janet to the idea of using variables to write a recursive procedure for the spiral image which they have generated (Figure 2). They had been introduced to the idea of recursion in a previous session. In order to understand Sally and Janet's approach to the problem, the teacher first asks them to explain what they have constructed.

Teacher: So what have you done each time? You add on two each time do you?

Sally: No.

T: Ten, ten. Then you have twelve, twelve ... then you have fourteen, fourteen.... So that's the pattern is it?... So you need to put a little bracket around that bit.... That's sort of the unit ... the module.... So you need to write a program to do that first, where instead of "10, 10" you put a word.... So it can be any size ...

S: Side.

T: Well it can be any word you like.... Use a different word from "side".... Do that and then call me back and then I'll show you how to add two ...

Janet: OK. So first of all we've got to call the program something.... Call it "ten".... What are we going to call that number?

S: Two.

A few minutes later the teacher returns and finds that Sally and Janet are not doing what she intended them to do. She intervenes again (they were attempting to use the REPEAT command instead of writing a recursive procedure).

T: Why are you doing "REPEAT 10"?
J: It should be two.
T: What are you repeating?
S: Miss, the commands.

She again directs them to write a general procedure for a "part" of the spiral.

S: Two.
T: I just want you to write a little program to do that bit and then I'll show you how to.... Instead of "FD 10", put a name for it ...
J: "FORWARD 2" — is it ... dot, dot two like that?...
S: But how does it know what the FORWARD is?

Despite their uncertainty, the pupils succeed, without help, in writing a general procedure for a "part" of their spiral :

```
TO TEN :TWO
FD :TWO
RT 90
FD :TWO
RT 90
END
```

They are then helped by the teacher to make this into the following recursive procedure:

```
TO TEN: TWO
FD: TWO
RT 90
FD :TWO
RT 90
TEN :TWO + 2
END
```

The important points to observe from this excerpt are that:

- the spiral task had been given to the pupils with the intention of provoking a recursive solution
- the teacher tried to understand the pupils' method of solution before directing the pupils to write a recursive procedure
- the teacher did not allow the pupils to pursue the idea of using "repeat", although this could have been a possibility
- the teacher did not want the pupils to use the word "side" as a variable name because there was evidence that pupils were attaching too much meaning to this type of word

- Sally and Janet demonstrated their acceptance and their developing understanding of the recursive procedure in the subsequent session when they generalized the angle and added a conditional statement to their procedure

At the beginning of the Logo Maths project, we planned to intervene in ways which would keep the pupils in control of their learning. We developed categories to analyse these interventions (see Chapter 9 of Hoyles and Sutherland, 1989). When we analysed the transcripts, we found that there were many episodes in which the interventions were very much in the control of the teacher, as the following excerpt taken from the lesson immediately before the Spiral lesson illustrates. This is the lesson in which Sally and Janet are introduced to the idea of writing a recursive procedure for the first time[1].

The Row of Pine Trees

Sally and Janet have written a procedure to draw a variably sized pine tree (Figure 3a) and have used this in "direct mode" to draw a row of pine trees (Figure 3) by entering the commands:

MOVE1, PINE 120, MOVE2, PINE 110, MOVE3, PINE 100 etc.

They want to write a procedure which will draw the whole picture and Janet does not want to type all the commands again. She requests help from the teacher.

J: Instead of typing all this out.... How are we going to make a big program?
S: But we can't 'cause we can't just type "REPEAT MOVE1 PINE 120", 'cause it's just going to keep on the same one all the time, isn't it?
J: Yeah, but ... yeah, I know, but is there any way we could do ...? No, I didn't think so.
S: Miss, is there any way you know how you do REPEAT it, so it won't do the same thing all the time?

Sally has requested help from the teacher, who sees this as a cue to introduce the idea of recursion.

S: Two.
T: Now, so imagine you're here at the end and you want to do your row. Now what you want to do is you want it to do PINE with a value, say, SIDE and then you want it to do MOVE2 and then you want it to do PINE again with — what's the next value?
J: One hundred and ten.
T: If the first one was PINE the value of SIDE, what's the value of the next PINE?

S: Minus ten.

T: Minus ten from SIDE. So if you'd done 110 from SIDE, then you do PINE again with minus ten from SIDE.... So what you want to do is to do PINE with a value of SIDE and then you do MOVE and then you want to subtract ten off the side.... Now do you see what that does? First of all, it does PINE with the value of SIDE you put in it. You move, and then it does it all again.... FOREST, now what does FOREST do? It does PINE with a value of ten less and it does MOVE. Then it gets to the bottom of here and it calls another copy of itself. So it does PINE — ten less. So it's going to go on and on and on and on. Do you understand? This is called recursion. So it starts and then it does itself again, but instead of having SIDE, it does it with ten less than SIDE. (Hoyles and Sutherland, 1989, p. 156)

The teacher then shows them how to write the recursive procedure FOREST (Figure 3b).

```
(a) TO PINE :SIDE        FD :SIDE
    LT 30
    BK :SIDE * 0.25
    FD :SIDE * 0.25
    RT 60
    BK :SIDE * 0.25
    FD :SIDE * 0.25
    LT 30
    BK :SIDE
    END
```

```
(b) TO FOREST :SIDE
    PINE :SIDE
    MOVE2
    FOREST :SIDE - 10
    END
```

Figure 3. The row of pines.

The difference between this excerpt and the Spiral excerpt is that in this session the teacher provides the pupils with the complete recursive solution together with an explanation of what she is doing. The teacher takes more control of the programming process than was the case in the Spiral episode. However in both sessions, the pupils have worked on the problem themselves and constructed "part-solutions" before the teacher intervenes.

Our categories were not appropriate for analysing this type of teaching episode, although we reported their occurrence (Hoyles and Sutherland, 1989). They did not fit with our model of "good" teaching and the implicit theoretical rationale for the study. We concluded that it would have been better if the teaching episodes had not been so prominent: "We now believe that it would be preferable to be more explicit about what we aim for the pupils to learn from a series of Logo sessions and plan 'teacher devised' tasks specifically to achieve these learning outcomes. Such tasks would reduce (though not eliminate) the need for teaching episodes and would allow pupils to retain control of their learning under teacher guidance" (Hoyles and Sutherland, 1989, p. 156). At the time, we were not conscious of the fact that our under-emphasis (from the point of view of analysing the data) of these episodes was related to our implicit ideas about teaching. This suggests that it is crucial to make assumptions about teaching and learning explicit at the beginning of a project so that results can be analysed and interpreted in the light of these assumptions.

At the end of the Logo Maths Project, we found that all eight pupils had learned to use variables in Logo to write general procedures for a simple geometric object (see Table II). Pupils' variable use in Logo has been categorized[2] as:

- variable operated on — a general relationship between variables within a procedure is made more explicit by operating on one or more variable inputs in a procedure (see, for example, Figure 4b)
- variable as scale factor — a variable input is used to scale all the distance commands in a "turtle" geometry procedure (see, for example, Figure 3a)
- more than one variable — variable inputs are added to a general procedure for each "object", which varies and thus any relationship between them is not made explicit
- no variables

Our analysis showed that the idea of variable had been the focus of considerable teaching intervention. This contrasts with our findings related to the learning of angle. We thought that pupils would learn about angle naturally for themselves by interacting with the Logo environment and consequently we intervened hardly at all on this topic. Analysis of the final interviews with the case-study pupils showed that pupils had learned very little about the idea of angle from their Logo experiences. We also found that on average we intervened less with pupils who were low achievers in mathematics than we did with the pupils who were high achievers in mathematics. These pupils, who were achieving less in their normal mathematics lessons and were receiving less teacher intervention in Logo, were also less competent at using variables in Logo at the end of the Logo Project. It was generally believed that too much teacher direction could inhibit learning. Nevertheless, it was found that those pupils who were involved in "more" teacher intervention learned "more" about the idea of variable. I am not suggesting a causal relationship at this stage but I believe that this finding needs more careful research.

TABLE I

Overview of Logo Projects

	Approx. No. of hours of Logo	No. of pupils studied	Age of pupils	Type of class	Teaching situation
Logo Maths Project	60	8	13 – 14	Mixed ability Comprehensive School	1 teacher working with a pupil pair
Peer Group Project	10	17	12 – 13	Mixed ability Comprehensive School	1 teacher working with 8 pupils
Project AnA	9	24	11 – 12	Mixed ability Comprehensive School	4 teacher working with 24 pupils

TABLE II

Pupil's competence at using variable in Logo:
A comparison of three research projects

	Proportion of pupils who operate on a variable	Proportion of pupils who use variable as scale factor	Proportion of pupils who use unrelated variables	Proportion of pupils who do not use variables	Total number of pupils
Logo Maths Project	$\frac{4}{8}$ (50%)	$\frac{2}{8}$ (25%)	$\frac{2}{8}$ (25%)	$\frac{0}{8}$ (0%)	8
Peer Group Project	$\frac{12}{17}$ (71%)	$\frac{0}{17}$ (0%)	$\frac{1}{17}$ (6%)	$\frac{4}{17}$ (23%)	17
Project AnA	$\frac{19}{24}$ (79%)	$\frac{0}{24}$ (0%)	$\frac{2}{24}$ (8%)	$\frac{3}{24}$ (13%)	24

FINDING AN EXPLANATORY THEORY

Results of the Logo Maths Project showed the need for an appropriate theory of teaching. Many psychologists and social psychologists are now turning to the work of Vygotsky (1962, 1978) and this is reflected in the research on mathematics education (see, for example, Hoyles and Noss, 1987; Nunes, Light, and

Mason, 1991). Crucial to Vygotsky's work is the idea that individual cognitive development results from social interaction in the world and that speech, social interaction, and co-operative activity are all important aspects of this social world. The active involvement of the child is a crucial part of the process, and in this respect Vygotsky's theories are similar to those of Piaget. Vygotsky differs from Piaget in his views about the role of speech and, more generally, symbol systems in cognitive development.

> The students of practical intelligence as well as those who study speech development often fail to recognize the interweaving of these two functions. Consequently the child's adaptive behaviour and sign-using activity are treated as parallel phenomena — a view that leads to Piaget's concept of "egocentric" speech. He did not attribute an important role to speech in the organization of the child's activities, nor did he stress its communicative functions, although he was obliged to admit its practical importance. Although practical intelligence and sign use can operate independently of each other in young children, the dialectical unity of these systems in the human adult is the very essence of complex human behaviour. Our analysis accords symbolic activity a specific organizing function that penetrates the process of tool use and produces fundamentally new forms of behaviour. (Vygotsky, 1978, p. 24)

Vygotsky also stresses that developing an understanding of the relationship between sign and meaning does not result from discovery by the child — the teacher has an important role in this process. Vygotsky differs from Piaget in that he specifically addresses teaching. He says that Piaget's emphasis on the child's spontaneous thought processes suggests that "child thought must be known as any enemy must be known in order to be fought successfully" (Vygotsky, 1962, p. 85). For Vygotsky, both spontaneous and non-spontaneous thought are intertwined. What is important is what the child can do with instruction, which must "be aimed not so much at the ripe but at the ripening fruit" (1962, p. 104). This resonates well with how we taught pupils to use variables in Logo, as opposed to how we thought we taught during the final two years of the Logo Maths Project. On reflection, it seems that until this point (that is, during most of the first year of the Logo Maths Project), we were waiting for development to occur before teaching, in that we were waiting until pupils chose projects which "needed" the idea of variable.

Vygotsky uses the term "the Zone of Proximal Development" to describe "the distance between the actual developmental level as determined by individual problem solving, and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). "Scaffolding", a metaphor first used by Wood, Bruner, and Ross (1976), describes the idea of simplifying the pupils' role whilst solving a task by means of graduated assistance from the adult/teacher. It is important to stress that this does not imply simplifying the actual task, but that the adult remove some of the cognitive demands. Scaffolding could take a number of different forms — for example, the adult could act as a memory bank for the pupil, could direct the pupils' attention, or could motivate and encourage the pupil to keep going. The adult participates in a certain sense in the solution of the task. This is substantially different from the idea that pupils should construct everything for themselves and also from the idea that pupils must be given "simpler" problems until they are ready for more difficult ones. The adult does not shy away from giving pupils challenging problems but somehow assists pupils whilst they are solving them.

In the Logo Maths Project, pupils were confronted with the idea of using a variable to write a general procedure within a number of "teacher-devised" tasks, and all eight pupils were eventually able to use the idea unassisted. It seems that our teacher interventions related to using variables in writing general Logo procedures were helping pupils to move from assisted to unassisted performance. If we take the two teaching episodes discussed in this paper, we see that when the pupils were first introduced to the idea of a recursive procedure (Row of Pines Task), the teacher essentially solved the problem for them, wrote the recursive procedure whilst providing explanation of the process. The pupils were subsequently given the Spiral Task which was again aimed at provoking the idea of recursion. Assistance was again provided, but during this session there were more gaps between teacher interventions and pupils were expected to construct more for themselves. Tharp and Gallimore have pointed out that "the shifting of goals by the adult to achieve intersubjectivity is the fundamental reason that a profound knowledge of subject matter is required of teachers who seek to assist performance. Without such knowledge, teachers cannot be ready to promptly assist performance, because they cannot quickly reformulate the goals of the interaction; they cannot map the child's conceptions of the task goal and the superordinate knowledge structures of the academic discipline that is being transmitted" (Tharp and Gallimore, 1991, p. 50). This profound knowledge involves both knowledge of the subject matter and knowledge of the pupils' conceptions and previous experiences. This was the case during the Logo Maths Project. The teacher/researcher observed all of the case study pupils' Logo sessions and was involved in analysing transcripts of these sessions. The teacher/researcher developed a very intimate relationship with the pupils, from the point of view of their learning about variables in Logo.

PEER GROUP DISCUSSION IN A COMPUTER ENVIRONMENT

Findings from the Logo Maths Project and a developing awareness of Vygotsky's theories prompted a different strategy of teacher intervention in subsequent projects. In the project "Peer Group Discussion in Mathematical Environments" we planned a sequence of sessions related to the idea of operating on a variable (see, for example, Figure 5), and the teacher/researcher made a conscious decision to emphasize this idea during the teaching sessions. One aim of the study was to investigate pupil discussion[3] as they solved a Logo task (Figure 4a). Pupils needed to be able to operate with variables to solve this task satisfactorily. The results of this study from the point of view of pupils' learning of the concept of variable show that after less "hands-on Logo time" more pupils were competent at operating on a variable than had been the case in the Logo Maths Project (see Table II).

```
TO HEAD :S
RT90
FD (:S/2)*1.5
LT90
FD :S
LT 30
FD(:S/8)* 9
LT 90
FD (:S/4)*7
LT 90
FD (:S/8)*9
LT 30
FD :S
END
```

Figure 4a. Pupils' solution to Head task.

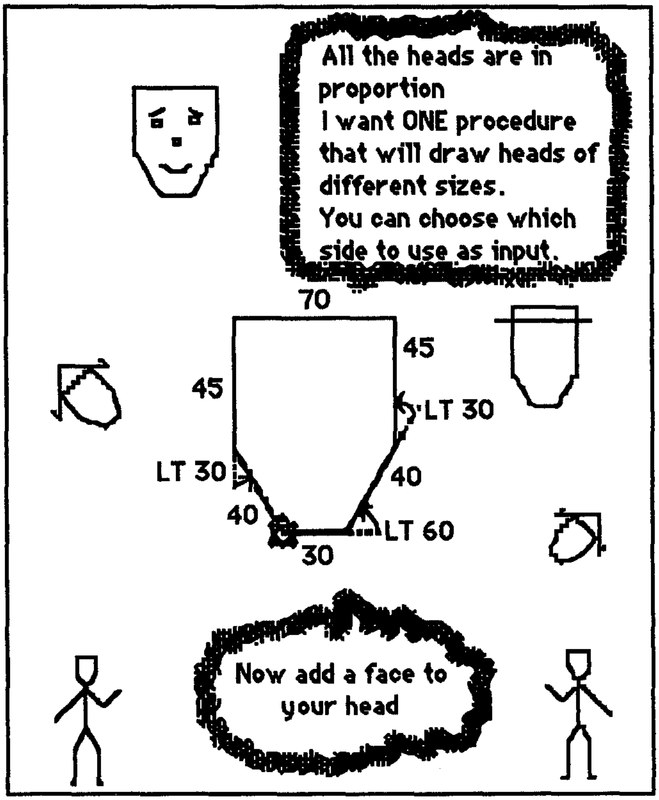


Figure 4b. Head task.

Results from both the Logo Maths Project and the Peer Group Discussion Project suggest that the teacher's use of language and the Logo language helped to structure the pupils' thinking when constructing a general Logo procedure (Healy, Hoyles, and Sutherland, 1990). This relates to Vygotsky's idea that speech plays an organizing role in pupil activity. We can find, in the transcripts, many examples of symbolic Logo code's being incorporated into natural language, as illustrated below:

Jessica: I know but I can't remember how you work it out, like if FORWARD 30 would be dot, dot S times something, and all, you know like that....
Anna: Well, what does 70 ... how 70 ... divided into 40 ...
J: Half of 70 is 35.
A: Thirty-five ... so it's 35, plus 15, dot, dot S, plus.... That's not right...

After an intervention from the teacher to stress that they need to work out the relationship between 70 and 30, Anna and Jessica continue their discussion.

A: Minus 40 ... no minus —
J: Yeah, minus 40.
A: I was right the first time ... but you don't use minus.
J: Well, add it ... something like divided or times.... If 70 is dot, dot S ... then it's, it must be divided by. Can't we say....

They carry out the calculation 70/30 on the computer and construct the Logo command in their general procedure:

```
FD :S/2.33333333
```

In the above extract, Jessica and Anna use the variable name "S" as they construct a general relationship. This variable name first plays a role as they communicate their ideas to one another and then plays a role as they communicate their ideas to the computer. The symbol "S" has taken on a communication function which fits well with the theory of Vygotsky.

PROJECT ANA

In the ongoing Project AnA, there is also an explicit aim to teach pupils to operate on a variable[4]. Pupils aged 11 and 12 are presented with the idea of variable within a carefully sequenced set of activities (see, for example, Figures 5 and 6b).

The emphasis is on operating on a variable to express a simple mathematical relationship; the geometric and arithmetic aspects of the introductory problems are intentionally de-emphasized. In this sense, pupil performance has been assisted by removing some of the cognitive demands of the task. This is done by carrying out an conceptual analysis of the mathematical, programming, and psychological aspects of the task. There is now enough previous work on pupils' use of variable in Logo for this to be possible. It would not have been possible to carry out this kind of analysis at the beginning of the Logo Maths Project,

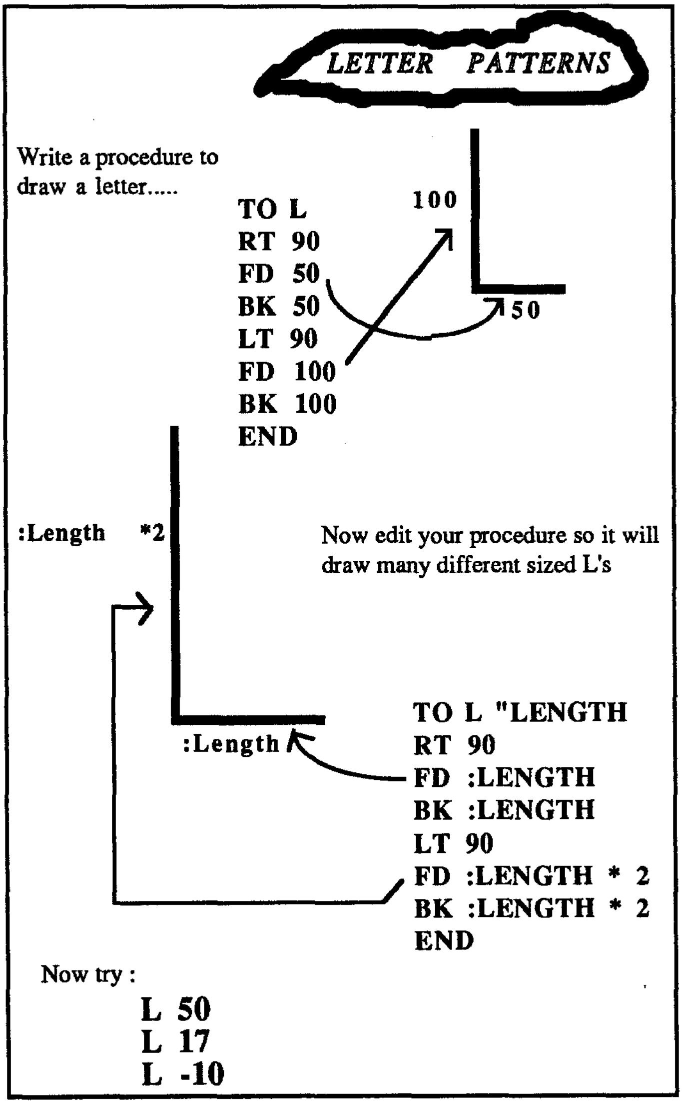


Figure 5. The "L" task.

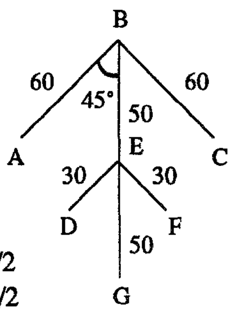
because little was known at that time about the cognitive demands of programming in Logo. If, at that point, we had attempted to prepare the type of teaching sequence used in Project AnA, we would have relied too heavily on research from non-computer settings. There is now increasing evidence (Hoyles and Noss, 1987; Sutherland, 1990; Tall, 1989) that there is a difference between pupils' performance in computer and non-computer settings.

In Project AnA, the Logo sessions are part of the pupils' regular mathematics lessons, and the whole class works in the computer room for one lesson a week. There are often as many as four[5] teachers in the computer room. There has been no attempt to standardize the teacher interventions, and the teachers are free to offer pupils support when they appear to need it. The situation imposes a structure on the teacher interventions in that pupils are actively engaged in their computer work and the teacher interventions are usually aimed at interacting with individual pairs of pupils working at the computer.

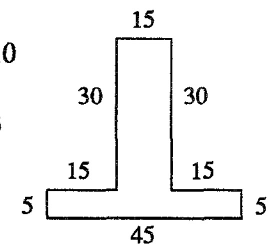
The discussion in this paper centres around the assertion that there has been a steady improvement in pupils' ability to operate on a variable in Logo from the first, to the second, to the third project (see Table II). In each project, performance has been assessed when pupils are working in pairs at the computer on a task which involves writing a general Logo procedure for a geometric object. In Project AnA, a higher proportion of pupils (19/24) are able to operate on a variable when constructing a general Logo procedure for a geometrical object than was the case in the Logo Maths Project (4/8), and the pupils in AnA are two years younger than the pupils in the Logo Maths Project. In addition, pupils in the Project AnA have had substantially less hours of hands-on experience and less individualized teaching than those in the Logo Maths Project (see Table I).

Together with this quantitative improvement, there has also been a qualitative improvement. Pupils in Project AnA (operate on a variable) were able to express more complex mathematical relationships (see, for example, Figure 6b) than the pupils in the Logo Maths Project (see, for example, Figure 6a).

```
TO HILL :JACK
    RT 90
    BK :JACK
    RT 45
    FD :JACK
    BK :JACK
    LT 90
    FD :JACK
    BK :JACK
    RT 45
    FD :JACK
    RT 45
    FD :JACK/2
    BK :JACK/2
    LT 90
```




```
TO MADDY :F
    BK :F * 5
    FD :F*10
    RT 90
    FD :F * 3
    RT 90
    FD :F * 10
    LT 90
    FD :F * 3
    RT 90
    FD :F
    RT 90
    FD :F * 9
    RT 90
    FD :F
```



```
FD :JACK/2
BK :JACK/2
RT 45
FD :JACK
END
```

Figure 6a. Arrowhead task.

```
RT 90
FD :F * 3
END
```

Figure 6b. Mad Hatter task.

My conjecture is that these improvements can be attributed to:

a) a higher expectation of pupil performance in Project AnA — freed from former, limiting expectations. In the Logo Maths Project, the idea of operating on a variable had been considered a conceptually difficult idea for pupils. This was not the case in the subsequent two projects when pupils were explicitly faced with the idea in a teacher-directed task (Figure 5);

b) a more carefully refined sequence of teaching activities, based on a conceptual analysis of using variable in Logo — exploration and constructing are important parts of this sequence;

c) a more relaxed approach to teacher intervention, within an overall atmosphere and expectation that pupils must take active responsibility for solving problems at the computer.

These conjectures need further refining, in particular (c) which relates to the finding that "more" teacher intervention on the use of variable seems to lead to "more" learning.

Another important finding from these three projects is that pupils' unassisted use of variables is strongly related to their first assisted use of the idea. In the Logo Maths Project, pupils were introduced to the idea of variable in the context of scaling a distance command, and some pupils continued to use variables in this way (see, for example, Figure 3a). Pupils were not introduced to the idea of scaling in the subsequent two projects, and no pupils from these projects used variables in this way (see Table II). In addition, in the Logo Maths Project we initially discouraged pupils from using single letters for variable names. This was not the case in the subsequent two projects. Many more pupils used single-letter names (e.g. a, x) in the two subsequent projects than was the case in the Logo Maths Project. There is no evidence that these "algebra-like" variable names present pupils with additional difficulties in Logo.

CONCLUSIONS

The main conclusion from this re-analysis of past research projects is that we need to make more explicit the underlying theories influencing our work. A denial of the importance of the teacher in the early days of the Logo Project influenced the way in which we categorized the data but did not influence the ways in which we worked as teachers. So there was a split between what we did and what we thought we did. Fortunately our methodology was open enough to allow us to re-examine these issues. The more rigid the experimental design, the less possible it is to reinterpret results in the light of new theoretical considerations. I suggest that we are still at the stage in educational research where there needs to be an interplay between theory and practice and this, I believe, implies a need for detailed process data on interactions between teachers and pupils. This is the type of research which is carried out by the French group of researchers and which, for practical reasons, is both difficult and time-consuming (Balacheff, 1990).

Vygotsky's theories point to a way forward but there are still a vast number of unanswered research questions. As Vygotsky says, "Practical experience also shows that direct teaching of concepts is impossible and fruitless. A teacher who tries to do this accomplishes nothing but empty verbalism, a parrot-like repetition of words by the child, simulating a knowledge of the corresponding concepts but actually covering up a vacuum" (Vygotsky, 1962, p. 83). This view is also reflected in what Brousseau calls the didactical paradox: "Everything he [the teacher] does to make the pupil produce the behaviours he expects tends to deprive the latter of the conditions necessary for understanding and learning the language concerned: if the teacher says what he wants, he can no longer obtain it" (Brousseau, 1984, p. 113).

NOTES

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[1] It is Sally and Janet's 14th Logo session, at the end of their second year of secondary school (aged 12 and 13). They have used the idea of variable to construct a simple geometric shape in seven previous sessions. For a more detailed discussion of their work see Sutherland, 1987.

[2] See Sutherland 1987 and 1989 for a more detailed discussion of these categories.

[3] Within the Peer Group Discussion Project, we were investigating pupils' discussion, and transcripts were only made of sessions in which the teacher did not intervene. Eight pupils were case studied, and data on pupils' use of variables were collected for 17 pupils in the class.

[4] Project AnA is a longitudinal study of pupils aimed at investigating developments from a more arithmetical to a more algebraic approach to problem solving. The emphasis is on detailed inter-

views with individual pupils. Detailed notes are made of the teaching sessions so pupils' development can be related to their experiences. The teaching sequences are carefully planned, but there is no detailed data on the teacher's spoken language.

[5] Class teacher; special support teacher; computer support teacher; researcher.

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