SUBTERRANEAN STRUCTURES OF TECHNOLOGICAL TOOLS AND TEACHING ISSUES IN GEOMETRY

Loïc Geeraerts, Fabienne Venant, Denis Tanguay

Université du Québec à Montréal (CANADA)

Abstract

This article focuses on issues related to the integration of dynamic geometry software packages (DGSs) in the teaching and learning of geometry. Appeared in the 1980s, this type of software quickly began to be widely used in classrooms. However, there is still a gap between expected benefits and teaching practices actually observed using DGSs. We rely on the instrumental approach as a theoretical framework to analyze the conditions according to which the full potential of these tools is developed. Taking GeoGebra as an example, we argue that doing dynamic geometry requires addressing new constraints that do not really fall into the scope of Euclidean geometry. This induces pedagogical thinking from a new perspective. Moreover, to achieve pedagogical goals without being deflected by the software requires adaptability and sometimes, sharp technical skills, which should now be integrated in the professional development of teachers.

Keywords: geometry teaching and learning, dynamic geometry software, dragging, instrumental genesis, software integration.

1 INTRODUCTION

The issue of integrating technological tools into teaching came up early in the field of mathematics. Being shaped from the start by the proximity between mathematics and computer science, with activities related to algorithmic, programming and computation, the reflection soon got diversified thanks to technological evolution and software development opening new possibilities of visualization and of direct experimental work on mathematical objects (Artigue, 2011).

So among others, a significant research trend emerged towards Dynamic Geometry Software packages (DGSs) such as Cabri-géomètre, The Geometer's Sketchpad or GeoGebra (Laborde & Capponi, 1994; Arzarello et al., 2002; Restrepo, 2008, etc.). These software packages are now largely utilized in the classroom of the industrialized countries since the 1980s, and have a decisive influence on geometry teaching and learning. But as it is the case with many technological tools, there seems to be a gap between the foreseen benefits and the technological practices really set in the classroom (Lagrange, 2009). Indeed, “Teachers and teacher educators […] are searching for guidelines that foster successful integration of new media into teaching practice. Researchers […] have difficulties in providing evidence of improved learning with technological means, as well as in understanding the influence of technology on learning” (Kieran & Drijvers, 2006, p. 205). We posit that conceiving such ‘guidelines’ requires a deep knowledge of the interweaving between the tools and the geometrical and didactical structured contents they stem from.

2 SOME THEORETICAL ELEMENTS

2.1 Instrumental genesis

Leaning on the instrumental genesis theoretical framing (Rabardel, 1995; Artigue, 2002; Lagrange, 2005, etc.), our research program is aimed at analyzing the complex relationship between students, teacher, knowledge and tasks in technological teaching environments. The frame draws a distinction between the artefact, the technological object itself, and the instrument which is constructed from it by an individual using it in a given type of activities. We call instrumental genesis the progressive elaboration of the instrument through the combination of two intertwined processes:

- **instrumentation**, by which the individual adapts his or her own action to the artefact;
- **instrumentalization**, by which the individual adapts the tool through the development of techniques and usage schemes, these corresponding to invariant conduct prompted when facing a given type of tasks.
This instrumental genesis is done at several levels. For instance according to the framing, a computer is an artefact, and a software such as GeoGebra may be seen as a form of instrumentalization of the computer, through which the designers of the software have adapted the potentialities of the computer and its screen, with the aim of promoting a more efficient teaching of school geometry. But of course, understanding minimally how the software works is a requirement, and this comes under instrumentation:

- first for the teacher who has either to choose among classroom activities already available or to design his/her own, the instrumentation required being more important in the latter case, and being nevertheless necessary for the trials supporting the choices and the possible adaptations in the former case;
- then for the students, according to different degrees in relation with the amount of autonomy requested by the planned activities.

We will see that the cycles of instrumentation and instrumentalization don’t stop here. But before going further into details about DGSs, GeoGebra, and their particularities, we need to discuss their main feature, their main instrumentalization directed towards teaching, that makes geometry performed with them a ‘dynamic’ geometry.

### 2.2 Drawings, figures and the dragging mode

A geometrical figure is an ideal object defined by a collection of properties. The most common way of having access to a figure is to consider a set of graphical representations that we will call ‘drawings’. A (geometrical) drawing is necessarily an approximation and a depletion. An approximation because the line segments are theoretically without thickness, the circles are supposed to be perfect, etc. A depletion because it would require in principle an infinite number of drawings to represent one figure. The student must then understand that the graphical representation stands for an ideal and generic — i.e. universal — representative, without singularity other than the ones given in the ‘hypotheses’ through the wording of the task or problem. In short, students must acknowledge that their understanding, from basically perceptive must move to ‘theoretical’, that they are no more working with drawings but with figures (Parzysz, 1988; Laborde et Capponi, 1994).

The Dynamic Geometry Software packages are likely to promote such an understanding. Indeed, the ‘dragging mode’ allows a direct invalidation of any construction made by a student using a DGS, and based on a perceptive reproduction (of a given figure or of one of the student’s mental models). Let’s suppose for example that the four sides of a parallelogram have been plotted by eye at the screen, that the drawing of a parallelogram has been plotted. Then the dragging of point $A$ with the mouse leads to a deformation: the figure is no more a parallelogram!

![Figure 1. Point A is dragged in a parallelogram plotted ‘by eye’.](image)

On the other way round, if the construction is done with the geometrical tools of the software such as *Segment between Two Points*, *Perpendicular Line*, *Circle with Centre through Point*, etc., here specifically with the tool *Parallel Line*, then the student must identify the aimed properties and bring them into play in an assumed way in his/her construction. The constructed representation then remains a parallelogram, a figure-parallelogram, when the vertex $A$ is dragged with the mouse: see Fig. 2.

Moreover, the dragging, when the construction is correctly done, engenders ‘in real time’ numerous representatives of the aimed figure, then allowing the student to grasp all the possible drawings of the same figure.

---

1 This is far from being GeoGebra’s unique usage domain. As the name suggests, one can also do Algebra with GeoGebra. In this article, we will limit ourselves to geometry and even more specifically, to synthetic (or euclidean) geometry, i.e. geometry without coordinates.
3 DYNAMIC GEOMETRY

So this is the main idea behind dynamic geometry: constructing a unique figure satisfying a predetermined set of constraints, and then being able to deform it so that the invariance of certain geometrical properties related to these constraints becomes apparent. No need to repeat the construction several times to be capable of stating a conjecture, or of verifying experimentally such or such property. The ‘dynamic figure’ then becomes the unique representative of the infinite set of possible configurations under the constraints at stake: a unique construction instead of multiple paper-and-pencil drawings.

3.1 Dynamic geometry and usage schemes

Yet as is the case when using the classical geometrical tools such as the ruler, the compass, the set square, etc., it is necessary to develop usage schemes with the software so to be able to construct dynamic figures. In the same way that a student doesn’t truly construct a perpendicular if he uses solely his ruler or misuses his set square, he doesn’t construct a ‘true’ perpendicular if he uses only the Line and Segment tools from a DGS to produce by eye a perpendicular. But while in the first case this type of behaviour is very difficult to detect unless the teacher benefits from a direct observation of the student drawing on paper, in the second case the dragging mode of the software allows a direct and ‘teacher-free’ feedback. When the construction ‘withstand to dragging’ i.e. keeps its aimed properties even if distorted by dragging, then the student has adequately taken the initial constraints into account and has really gained access to instrumented geometry, if not to theoretical geometry.

3.2 An extra layer over classical geometry

Unfortunately, correctly using the geometrical tools of a DGS is not sufficient to produce a construction that will stay stable when distorted by dragging. A majority of students, among those who correctly construct the requested figures with paper-and-pencil, are facing new difficulties when carrying out dynamic constructions that should be absolutely stable whatever the deformation. This brings us to posit that dynamic geometry is something much more complex than classical school geometry with some extra facilities, but is rather a new geometry, resulting from the integration of an additional layer over classical geometry.

This new geometry may well have pedagogical and didactical virtues, it also increases the geometrical complexity, a state of affairs that teachers must be aware of if they are eager to avoid misunderstanding and ‘breach of contract’ with their students. Let’s illustrate this through a first example. The task is classical and could be traced back as far as the ancient Greeks: the “ruler and compass” problem of tracing the perpendicular line to line AB through point M outside AB. We will examine the equivalent problem in dynamic geometry, which consists in tracing the perpendicular line through M but without using the Perpendicular Line tool. The idea here is to bring students to carry out a construction uniquely based on properties linked to the notion of equidistance. Fig. 3a) and 3b) below are reproduced from GeoGebra screens.

The construction protocol may be the following:

1. With Circle with Centre through Point or with the tool Compasses, the circle $C_1$ of centre $M$ passing through $G$ is drawn. Point $G$ is an arbitrary point clicked with the mouse while tracing $C_1$. Points $I$ and $J$ are thus obtained as intersections between $C_1$ and line $AB$. They are equidistant from $M$, so that $MI = MJ$.

2. Circle $C_2$ of centre $I$ and passing through $M$ is drawn. Circle $C_3$ of centre $J$ and passing through $M$ is drawn. The second point of intersection between $C_2$ and $C_3$ is labelled $K$, the first being $M$. We then have four equal lengths $MI = MJ = KI = KJ$, so that $MIJK$ is a rhombus.
3. The diagonals of a rhombus being perpendicular, the diagonals \([IJ]\) and \([KM]\) are supported by the perpendicular lines \(IJ\) which is the same than \(AB\), and \(KM\), which is the line to be produced.

We may consider that a student having produced a similar construction has succeed in doing the task. So we may expect that the software validates the construction and that the dragging of either line \(AB\) or point \(M\) won’t affect the basic properties of the constructed figure.

But now a difficulty specific to the dragging mode comes into play. Indeed, by dragging point \(M\) away from line \(AB\), we may bring the intersection between circle \(C_1\) and line \(AB\) to disappear, and in its wake all the other elements of the construction depending on points \(I\) and \(J\), as shown in Fig. 3b). Hence we observe here that pure geometrical knowledge is not sufficient in dynamic geometry. In this typical example, students must anticipate the movement resulting from dragging, must keep in mind which are the objects free to move in the construction and what is going to happen if these objects are moved. Such considerations are of course absent from working with paper-and-pencil. In terms of instrumentation, it suggests that student will have to develop new strategies relying on specific knowledge pertaining to a new geometrical world.

For instance in this activity, the student must be conscious that using the tool Compasses or the tool Circle with Centre through Point induces the creation of a new point \(G\) (see Fig. 3). Since this point is not laid on line \(AB\), the circle \(C_1\) passing through \(G\) won’t ‘follow’ line \(AB\) when needed. This may cause points \(I\) and \(J\) to disappear, as shown in Figure 3b). If we want to ‘force’ circle \(C_1\) to keep a non empty intersection with \(AB\), we have to add one more constraint and define point \(G\) to be on line \(AB\). We note that the only difference with the first construction is then that points \(I\) and \(G\) (or \(J\) and \(G\)) are now the same.

4 GEOGEBRAIC GEOMETRY

Now that we have seen one of the main difficulty pertaining to dynamic geometry, one may wonder if there exists notable differences between distinct dynamic geometry software packages. We will examine this question by considering a fundamental phenomenon in mathematics and particularly in geometry, that is proportionality of measurements.

4.1 A misleading genericity

As we mentioned in § 2.2, one of the interest of dynamic geometry is the possibility of working with a generic figure and of exploring ‘infinitely’ many cases simply by dragging points and distorting the figure. But we claim that with certain DGSs, unexpected proportionality properties appear that neither the user, nor the rules and laws of Euclidean geometry have asked for. We illustrate this claim with the right-angled triangle. Fig. 6 below shows a right-angled triangle constructed with GeoGebra and its tool Perpendicular Line.
Fig. 4. Right-angled triangles constructed with GeoGebra and their measurements.

We may observe that when point \( A \) or point \( B \) is dragged, point \( C \) moves as if the triangle was subject to a dilatation or a contraction, that is with lengths of sides \( AB \) and \( BC \) keeping always the same proportion. Yet in these conditions, since the construction of the triangle is done without anything specific other than the tool \textit{Perpendicular Line}, we should expect a generic right-angled triangle. An attentive student would be entitled to conclude that all right-angled triangles have their two short sides in a unique and same relation of proportionality, which is of course completely erroneous. More generally, the problem raised by this type of software programming is that the figure supposed to represent all possible right-angled triangles in the world are in fact restricted into a unique class of similar triangles.

From the teachers’ perspective, it thus appears to us that being conscious of this kind of problems is fairly important, in order for instance to avoid the appearance of misconception in students’ mind. We analyse the increase of such an awareness as resorting to instrumentation. But an expert in GeoGebra is capable of going further, by manoeuvring a fine instrumentalization of the software that would consist in creating a new tool \textit{Generic Perpendicular Line}. It is about getting around the pitfall by constraining the construction of right-angled triangles genuinely generic see Fig. 5. Such an instrumentalization relies on advanced knowledge about the functionalities available in GeoGebra and about the ways GeoGebra operates with the different geometrical objects.

Fig. 5. Two non similar right-angled triangles constructed with a new tool in GeoGebra.

4.2 Experimenting and conjecturing with GeoGebra

One of the recognized didactical function of the dragging mode is to allow the conjecture and the experimental verification of a geometrical theorem. A figure is constructed according to the hypotheses of the theorem and a great number of cases are explored or verified by dragging. We illustrate this practise with the theorem that states that the sum of the three angle measurements in a triangle always equal \( 180^\circ \).
The idea is to bring students to conjecture the result through the exploration of cases generated by dragging one of the vertices of a triangle whose angle measurements are also made apparent at the screen: see Fig 6. In our view, the activity keeps its interest if the approximate, non exact measurements are exploited to bring students to the necessity of going beyond experimental verification, and of asking for a (formal) proof of the statement.

Once the students are aware that the three measurements must be added, instead of using their calculator or mere paper-and-pencil computation, they can ask GeoGebra to do automatically the sum by entering the symbolized sum into the input box. But that’s where things get messed up. First the result given in the box (see Fig. 6) is always 180°, which is problematic if we want the truth stake to be kept alive (Grenier & Payan, 1998). Secondly, if we ask the software for more decimal places, we may realize that there is a discrepancy between the result given in the box and the measurements, as numbers displayed at the screen. In Fig. 6, we can see by simple inspection of the last digit in each number that the sum is not exactly equal to 180, whereas 180 is systematically given as a result in the sum box. This may be a good thing in the perspective of promoting the necessity for a proof. But if students ask for explanations regarding the discrepancy, the teacher has to get into technical considerations about how the software internally stores, processes and displays the numerical data. It may of course be interesting and informative, but there is a real threat that the initial pedagogical goal of the activity gets lost of sight.

Another possible response from the teacher could consist in the following instrumentalization: constraining GeoGebra to add the displayed numbers instead of the numbers internally stored. Going even further, we may as well control the number of decimal places by using sliders directly reachable on the screen (see Fig. 7), without the student having to know where to adjust the number of decimal places in the meandering of the software menu. Thus, many instrumentation problems are settled at a time. Not only the consistency of the numerical displays is recovered, but mainly the truth stake comes back to the fore in students’ mind.

---

2 For more technical and didactical details, see Tanguay & Geeraerts, 2014.
5 CONCLUSION

Although most of the individuals and communities involved in the field of education (institutions, software and content designers, parents, society...) increasingly encourage the use of technology, and more specifically of dynamic geometry software packages, GeoGebra being one of the most widespread, while the importance of underlying obstacles maybe minimized or kept unnoticed, we must be aware of the inherent difficulties in the use of these tools. Their usage is based on an instrumentation far from being obvious, neither for students nor for teachers. Teachers should indeed be conscious that doing dynamic geometry requires addressing new constraints that do not really fall into the scope of Euclidean geometry. It calls for pedagogical and epistemological reflections, these in turn calling for specific and dedicated objectives in teacher training, whether pre-service, in-service or both.

Furthermore, it is important to be aware that choosing a software rather than another can lead having to face behaviours that are unexpected, even questionable from a mathematical standpoint. For teachers, it means being able to control them by adequate instrumentalization, or to adjust classroom planning so to avoid unwelcome inconsistencies. We exemplified such behaviours from GeoGebra with the “Perpendicular Line” tool and the management of rounding in (angle) measurements. Fine instrumentalization is thus necessary if one wants to achieve his/her educational objectives, without being deflected by the software. It is based on adaptability and sometimes need sharp technical skills. Considering the increasing place occupied by these new technologies, the acquisition of such skills should be considered when teachers' professional development is examined and discussed.

REFERENCES


