ENACTIVIST SIGNIFICANCES FOR MATHEMATICS EDUCATION RESEARCH

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Building on previous PME work, we continue the conversation about the significance of the enactivist theory of cognition for mathematics education research. To do so, we raise a number of distinctions that are to be conceived of as possibilities and invitations to understand what an enactivist entry can offer.

INTRODUCTION

At last year's PME, we presented a number of distinctions between constructivism and enactivism that we believed were fruitful to understand their differences as theories about (mathematical) cognition (Proulx & Simmt, 2016). Among others, three of the distinctions we saw as differences that could make a difference were relative to a transition: from a focus on (prior) knowledge to one on doing; from an interpretive view of the world to one of bringing forth the world; and from problem-solving to problem-posing. Insisting on the importance of avoiding the dichotomy between the two theories, we concluded by initiating a reflection on some of the possibilities that enactivism offered for mathematics education research. This year, we directly address these issues and explore in depth a number of outcomes for mathematics education research that the enactivist sensibility brings forth.

AN INITIAL POSITIONING: ENACTIVISM IS NOT A RESEARCH LENS

The concept of lens, often used in research as an explanation of what researchers do when they "look at" data through a particular theoretical framework, does not work well for enactivism, because enactivism has been conceived as a way of being. Our work is inspired by Maturana's (1988a) theory of the observer, where the observer is central to any *account* of any phenomenon: "everything said is said by an observer to another observer that could be himself or herself" (Maturana, 1988a, p. 27). As we explain elsewhere (Proulx, 2016), even if researchers generally do not believe that "the phenomenon being observed" is a fact independent of the observer and that it can be decontextualized from the observational act, this position is often implicitly taken by how research findings are reported. Like Barwell (2007) asserted at PME-31, even if one agrees that one cannot account for what *really happens*, research is still being reported (and conceived) as if this were the case. Consider the view that analyzing from various perspectives can generate a deeper understanding of the phenomenon observed, or the claim that the complexity of the classroom cannot be *understood* only from one perspective but requires a variety of viewpoints. Enactivists are uncomfortable with this positioning, to the extent that it does not recognize that the "understandings" are of the observer's experience and not of the phenomena existing in itself: a multiple lens approach entails an assumption that "the phenomenon being observed" is a fact independent of the observer and can be decontextualized from the observational act, leading to the possibility of aspiring to an accurate account of it. For enactivists, reports of research findings are explanations for what is observed as the observer brings forth coherence in the world.

Thus, the issue of "accurate account" is meaningless if the researcher holds an epistemological position where one does not *describe* what is being observed, but brings forth one's own account of one's own perceptions (Maturana, 1988b). In our case as researchers, adequacy of mathematical actions is not linked to some objective referent, but to the eye of the observer who *assesses* it on the basis of his/her own set of criteria. This is particularly important for us as researchers who attempt to talk about students' mathematical activity: students' mathematical actions are in fact observations, *conceptualisations* made by us as researchers. Accordingly, "accurate accounts" takes another meaning, less about truth or validity and more about generativity, about distinctions and insights it offers us and to others. Enactivism is not about interpretations through a lens (i.e., about descriptions) but about bringing forth a world through engaging in that world, as Maturana and Varela (1987) express:

> We do not see the 'space' of the world; we live our field of vision; we do not see the 'colors' of the world, we live our chromatic space (pp. 22-23).

A VIGNETTE ON SOLVING ALGEBRAIC EQUATIONS

In one of our projects, Grade-8 students were asked to solve without pencil-and-paper usual algebraic equations for *x*. For $\frac{2}{5}x=\frac{1}{2}$, some of the students' strategies were:

Transforming in equivalent fractions and decimals. One student explained having transformed ½ in 10/20 to make the ½ divided by 2/5 simpler, then repeating the same thing for 2/5 to get $\frac{20}{50}x=\frac{10}{20}$. He explained it to be equivalent to $0.4x=0.5$ hence $x=0.5/0.4$.

Inversing and transforming in decimals. One student explained inversing the equation to $\frac{5}{2}x=2$.He then transformed in decimals, $2.5x=2$, and dividing by 2 got $1.25x=1$ so $x=1.25$.

Cross multiplying. After making the equation $\frac{2x}{5}=\frac{1}{2}$, the student explained having cross multiplied, where 5 times ½ gave $2x= 2.5$ and thus $x=1.25$.

Halving. One student explained that half of 5 is 2.5 and because one looks for ½, $x=1.25$.

Finding a scalar. One student explained looking for the value of *x* that made 2/5 equal ½. Placing fractions over 10, he explained that $x=1.25$ because 4 times 1.25 is 5 and 5/10=½.

Finding common denominator and adding. The student explained having placed fractions over 10, obtaining $\frac{4}{10}x=\frac{5}{10}$. Subtracting 4/10 and 5/10 gave –1/10, so then *x* is worth 1/10.

A group of teachers, some being the students' teachers, were also asked to solve, without pencil-and-paper, similar equations for *x* For $\frac{2}{5}x=\frac{1}{2}$, some strategies were:

> *Equating middle and extreme products*. One teacher explained having acted like with ratios, multiplying middle and extreme terms together, obtaining 4x=5, hence x=5/4.
>
> *Multiplying by the inverse*. One teacher explained having divided by 2/5 on each side of the equality, leading to multiply by 5/2 to get the same answer, giving x=5/4.
>
> *Isolating* x *in two steps*. One teacher explained having multiplied by 5 on each side of the equality, obtaining 2x=5/2, and then dividing all by 2 to obtain x=5/4.

THE EXPERIENTIAL SUBJECT

The enactivism maxim "all doing is knowing and all knowing is doing" invites a transition from a focus on (prior) mathematical knowledge to one about mathematical doings. This in turn leads to a move from the constructivist or psychological Kantian subject that builds or takes things in, toward more phenomenological experiential subject that (en)acts. Moving away from knowledge and toward doing is not without consequences for mathematics education, because it makes students' mathematical experiences central in the teaching-learning phenomena; in contrast to one about what students know, need to know or don't know. In that sense, the focus of instruction or of analysis of students' mathematics, to reuse Les Steffe's formulation, becomes altered. Concretely, this resonates with Seymour Papert's idea of enriching mathematical experiences of students, where the focus is on the richness or poorness of experiences students are exposed to – and not on the poorness or failures of the student *per se* (and teachers alike). Being experientially poor, or plunged in a culturally poor environment, is very different than being cognitively poor; this invites a significant change in focus for mathematics education (research).

This suggests a different focus for the analysis of students' work. In the vignette above on solving algebraic equations, we note how students' and teachers' strategies are different. Teachers' work is aligned toward standard, and more automatic, general strategies (a sort of mathematical "reflex") for solving algebraic equations. In the students, one sees more local solving steps, directly sensitive to the data in the equation to solve, whereas for teachers the strategies developed appear more decontextualized and general. Obviously students' experiences are quite different from that of a teacher. For teachers, one has the impression that their (teaching) experiences intensely orient their ways of solving, as one teacher expressed:

> I would show it like that to my students [step by step, operating on each side of the equation in the same manner]. […] However, as a secondary student I was never shown this "balancing" way. One day one of my colleagues told me "Listen, I teach it like that" and then when teaching Grade-7 I slowly started it and we always said "you do the inverse operation, bing, bang". And now I wonder if it has not become an automatism. Is it because we have done it so often like this?

Faced with having to teach to students, teachers make choices that in turn orient and influence the nature of their own mathematical experiences of a specific mathematics thematic. It is this experience that plays a major role for teachers and differentiates it from students. Teachers have become expert solvers and are able to perceive all these equations through the same algebraic lens, impressively steering almost all equations to the same sort of task. Students are novices in comparison, not having yet lived similar repetitive experiences that teachers have. One would not want to conclude that these students do not understand, have misconceptions, have partial knowledge, and so forth. Rather, enactivists wonder about the nature of the previous and immediate experiences of the student in the mathematics class: what sort of environment has the student been plunged into, what sort of activity governs this students' mathematical immersion? What sort of mathematics is this student doing and why are some actions being attempted? What is the mathematics being enacted?

This attention on experiences aligns well with what is understood from studies of mathematicians. In her work, Burton (2004) shows that it is not the mathematician's knowledge base that plays the critical role in how they solve problems, nor the possession of a toolbox of strategies to apply when problems require it. Rather, it is primarily about being able to do, to solve, to act, from their cumulative experiences. Papert (1972) is direct about how the focus on knowledge in school is misguided:

> Being a mathematician is no more definable as 'knowing' a set of mathematical facts than being a poet is definable as knowing a set of linguistic facts. Some modern mathematical education reformers will give this statement a too easy assent with the comment: 'Yes, they must understand, not merely know'. But this misses the capital point that being a mathematician, again like being a poet, or a composer or an engineer, means doing, rather than knowing or understanding. (p. 249)

EXPLAINING RESEARCHERS' OBSERVATION OF THE MULTI-VERSE

Maturana invites researchers to think about a multi-verse rather than a uni-verse, with this assertion about how different members of a family experience the family:

> Systems theory first enabled us to recognize that all the different views presented by the different members of a family has some validity, but systems theory implied that there were different views of the same system. What I am saying is different. I am *not* saying that the different descriptions that the members of a family make are different views of the *same* system. I am saying that there is no one way which the system is; that there is no absolute, objective family. I am saying that for each member there is a different family, and that each of these is absolutely valid. (Maturana, in Simon, 1985, p. 36)

His conception of the multi-verse suggests that rather than considering students' work in terms of a variety of interpretations of a single problem or task, we instead consider the varieties of mathematical worlds students bring forth with their activity; ones that are adequately valid from the point of view of the student bringing it forth.

In relation to the vignette about strategies for solving $\frac{2}{5}x=\frac{1}{2}$, this could mean that these do not represent a variety of interpretations of $\frac{2}{5}x=\frac{1}{2}$ from a variety of students, but rather that there are different equations being solved by each solver. Each student and teacher brings forth their own equation and show, by their strategy, that what appears to be for the observer the same equation is in fact not the same. This bringing forth opens to a variety of mathematical worlds, resulting in a multi-verse being brought forth. The fact that claims are made on those equations seem to suggest that they are from a world with transcendental objectivity (uni-verse), but as Maturana explains, we need to recognize that these claims are our explanations that arise in our distinctions of distinctions from our lived experiences as observers.

ERRORS AS OBSERVER'S DISTINCTIONS

In the constructivist era, questions of mathematical errors were highlighted as important elements to reconceptualise, departing from medical paradigms that viewed them as pathologies to diagnose and eradicate. Constructivism suggested errors were partial knowledge, in construction, and not failures. This led to studies and practices aimed at working with errors, learning from them, working with them. During this same era, Maturana offered that errors could be more than partial knowledge or under construction, and that they be seen as knowledge in themselves. For him, it is impossible for someone in the moment of the experience to distinguish between perception and illusion. This impossibility means that it is only from "outside" (observer's perspective) that something can appear to be more or less complete.

> All this is grounded on the facts that we live as valid whatever we live, and that in the experience of living whatever we live we do not know if we shall later treat what we have lived as an illusion or as a perception when we compare it with another experience of whose validity we do not doubt. (Maturana, in Maturana & Bitbol, 2012, p. 176)

For Maturana, we are coherent beings acting along this coherence (at least we live it that way). In the moment of enactment, students act coherently with what they know, trying to give meaning to the situation. In a word, students do not attempt to make mistakes (and if they do or succeed at it, then it is not a mistake anymore!). What this leads to is that there are no possible distinctions between errors and knowledge in the moment of its enactment, in the experience itself, and it is only in light of other experiences that this or that action or response takes the status of error or knowledge.

ETHICAL ISSUES STANDARD

Enactivism makes us more sensitive to the ethical stances invoked by the claims on errors. Asserting an error brings with it a comparison, one with the established body of knowledge, lying outside of one's experiences. Considering mathematical knowledge as an external thing "to know about" unfortunately leads to comparing students' mathematics with this external body of mathematics. In these comparisons, students are always seen as lacking something, as needing more to achieve this knowledge base. In a word, students and their knowing are seen in deficit terms.

This raises ethical concerns about the constraining of students' mathematical production within these boundaries of comparison. Comparing constrains researcher's (or teacher's) understanding of students' activity because what students do/understand/produce is always seen in comparison with this external mathematical knowledge base. Hence, it is always a subset of what *we* already know, and thus can barely be seen as mathematically creative. Borasi (1987) boldly asserts that this situation is representative of *our own* lack of creativity as researchers:

> […] the creativity of the researchers themselves when analyzing the error would be constrained by their limited focus on finding the causes of the students' error so that they could eliminate it. Thus they see the error necessarily as a deviation from an established body of knowledge, and do not even allow themselves to consider it as a possible challenge to the standard results. (p. 4)

By making comparisons between students' mathematics and this knowledge base, we are the ones creating the deficit in students' mathematics. Then we are surprised when faced with the situation that students lack knowledge; this should hardly be a surprise since the deficit is built into the comparison. Enactivism leads us to be sensitive to this ethical situation, asking us to think differently. Rather than focusing on knowledge, our focus is placed on action, on the doing of mathematics.

MATHEMATICS AS AN ACTIVITY ENMESHED IN THE WORLD

This focus on mathematical doings leads to conceptualizing mathematics as an activity. Mathematics as activity is not a set of concepts/information built up in a linear order: it is more about the recursive evolvement of a landscape than one of a building (see e.g., Ernest, 2016). This is akin to Davis & Hersh (1982) assertions about the active practice of mathematics, which produces mathematics that in turn influences future mathematical practices. Davis and Hersh offer the example of the sum of the measures of the angles in a triangle that amounts to 180º as a piece of mathematics that has been both maintained through time, and evolved through time:

> Archimedes knew this as a phenomenon of nature as well as a conclusion deduced on the basis of the axioms of Euclid. Newton knew the statement as a deduction and as application, but he might also have pondered the question of whether the statement is so true, so bound up with what is right in the universe, that God Almighty could not set it aside. Gauss knew that the statement was sometimes valid and sometimes invalid depending on how one started the fame of deduction, and he worried about what other strange contradictions to Euclid could be derived on a similar basis. (p. 33)

They give another example in relation to counting and arithmetic, done through the years in a variety of ways (e.g., using stones, abacuses, finger counting, slates carving, pencil-and-paper, adding machines and now computers). Davis and Hersh argue that "[e]ach of these modes leads one to a slightly different perception of, and a different relationship to, the integers” (p. 33). In short, mathematics is evolving and enmeshed in the cultures, eras, and people doing it.

In addition, this concept of evolving and enmeshment in the world and culture in which it takes shape connects mathematics directly to that world. Enactivists, by referring to circular dynamics (see Varela, 1984) asserts that the fit of mathematics with the world is trivial, since mathematics arose from our actions in the world. In that sense, the “discovered or invented” debate in the philosophy of mathematics does not suit enactivism well. Enactivism is centered on the enmeshment in the world, where mathematics co-emerges in that world. This focus on co-enmeshment is used to explain for example some people’s surprise in the great fit of mathematics within the world in which we live, leading them to assert that our world is mathematical and setting aside the idea that, as Maturana and Varela would say, we live our mathematical world. As de Freitas and Sinclair (2014) allude, mathematics is not a rigid, determined and constrained discipline: mathematics happens in the doing (of mathematics). Elsewhere, we used doing|mathematics, “an expression that alludes to the emergent made-up nature of both the doing and the mathematics and their dialectical relationship in lived-life mathematical experiences” (Maheux & Proulx, 2015, p. 214). In that sense:

> Doing|mathematics is both doing something (some thing) recognizable as mathematics, but also producing mathematics as this thing that we are doing when we do what we do. And although the two interpretations of the expression doing|mathematics are incompatible as fixed states, they echo the productive circularity at the heart of our enactivist thinking, according to which there is no objective end or start, but only an observational beginning. Both directions thus develop simultaneously, nourishing one another and drifting in the same current. (p. 215)

CONCLUDING REMARKS – ON CIRCULARITY

Enactivism offers a number of different possibilities for researchers through its suggestion of how we live the world. Those suggestions range from how we conduct and bring forth our research, from the observer’s perspective, to how mathematics itself is lived as a way of engagement within the world we bring forth. The enactivist reference to circular dynamics decries both positivist’s top-down view of objective/external knowledge and post-positivist assertions of subjective knowledge emerging from the bottom-up. The enactivist position (e.g. Thompson & Varela, 2001) cuts across both views and calls for a dialectical and dynamical conception that combines top-down and bottom-up distinctions, conceiving of them as continuously emergent phenomena that subsequently imposes themselves, through a never-ending recursive loop of production and constraint. This enmeshment leads to ethical concerns, for considerations of the never-ending evolvement with mathematics as we bring it forth living the world. In Varela’s (1996, p. 96) words, “knower and known, subject and object, are reciprocal and simultaneous specifications of each other. In philosophical terms: knowledge is *ontological*.”

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